Modellierung von Kern- und Wicklungsverlusten

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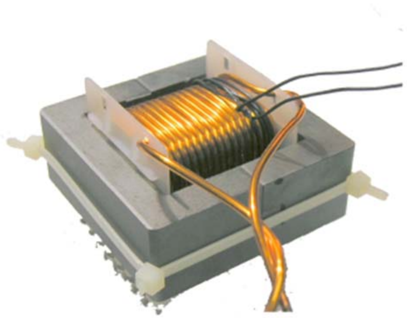
Motivation

Modeling Inductive Components



Employing best state-of-the-art approaches and

embedding newly developed approaches

into a novel loss calculation framework.

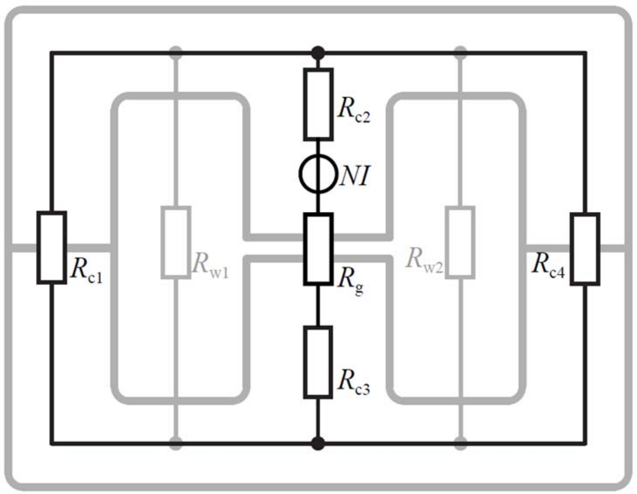
Loss Calculation Framework

Overview

1) **A reluctance model is introduced to describe the electric / magnetic interface, i.e. *L* = *f*(*i*).**

2) **Core losses are calculated.**

3) **Winding losses are calculated.**



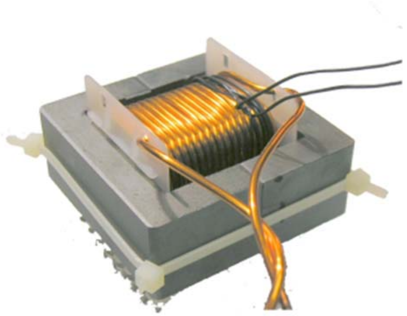
Agenda

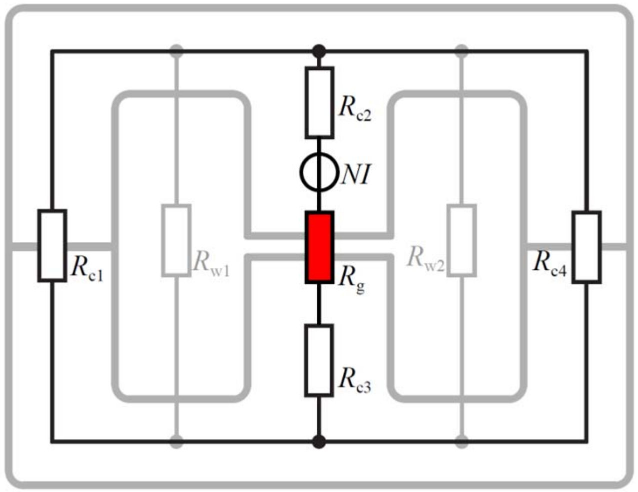
- Motivation
- Reluctance Model
- Core Losses
- Winding Losses
- Experimental Results

Motivation

Modeling Inductive Components

Calculation of the inductance *L* with a reluctance model





Inductance calculation

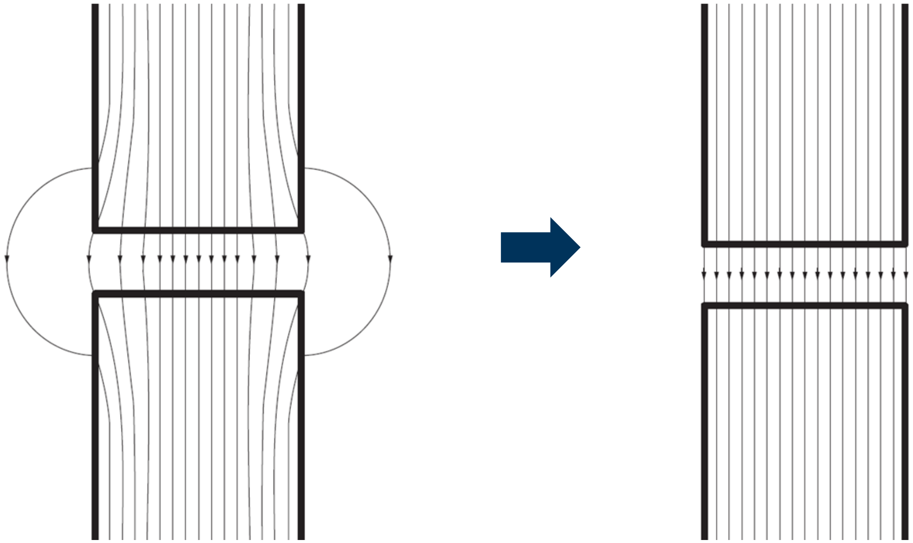
$$L = \frac{N^2}{R_{\mathrm{tot}}}$$

Reluctance definition

$$mmf = R_{\mathrm{m}} \Phi$$

Existing Calculation Approaches

Assumption of homogeneous field distribution



Air gap reluctance calculation

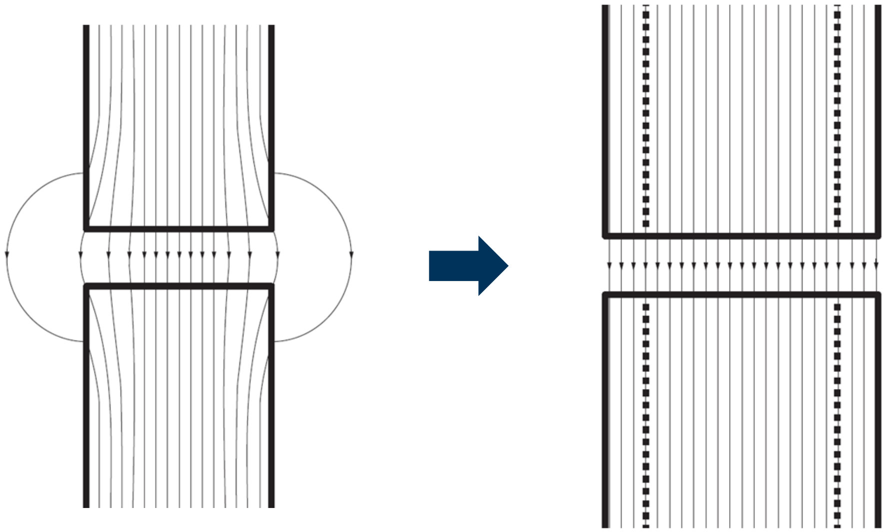
$$R_{\mathrm{m}} = \frac{l_{\mathrm{g}}}{\mu_0 A_{\mathrm{g}}}$$

l_{g} Air gap length

A_{g} Air gap cross-sectional area

Existing Calculation Approaches

Increase of the Air Gap Cross-Sectional Area



Air gap reluctance calculation

$$R_\mathrm{m} = \frac{l_\mathrm{g}}{\mu_0 A_\mathrm{g}^*}$$

l_g Air gap length

A_g^* "New" air gap cross-sectional area

e.g. [4] (for a cross section with dimension *a* x *d*):

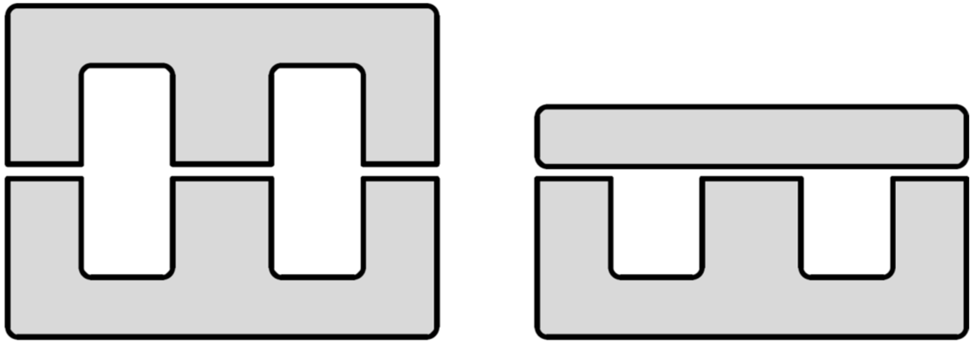
$$R_\mathrm{m} = \frac{l_\mathrm{g}}{\mu_0 (a + l_\mathrm{g})(d + l_\mathrm{g})}$$

Aim of new model

Air gap reluctance calculation that

- considers the **three dimensionality**,
- is reasonable **easy-to-handle**,
- is capable of modeling **different shape of air gaps**,
- while still achieving a **high accuracy**.

Illustration of different air gap shapes:



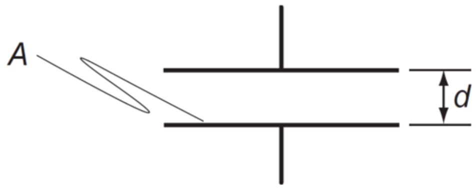
Derivation of new model

Capacitance-To-Reluctance Analogy

If air is the dielectric, capacitance *C* can be expressed as

$$C = \varepsilon_0 F(g)$$

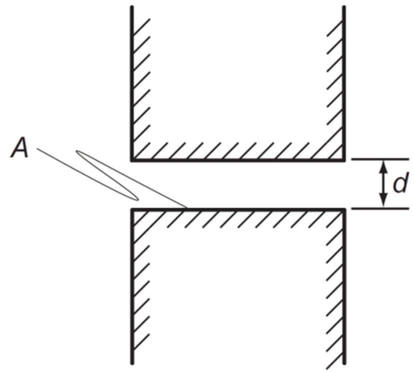
Represents geometry



$$C = \varepsilon_0 \frac{A}{d}$$

The reluctance of an air gap with the same geometry is then

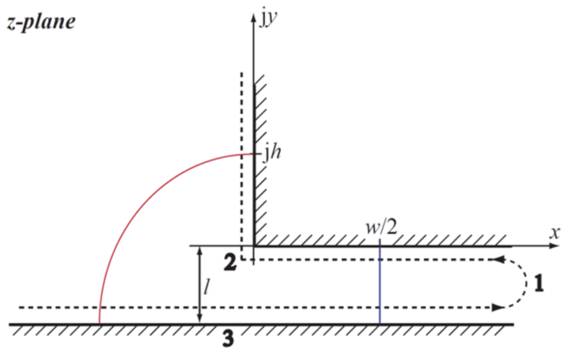
$$R_{\mathrm{m,airgap}} = \frac{1}{\mu_0 F(g)}$$

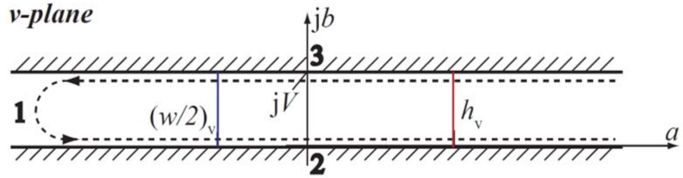


$$R_{\mathrm{m,airgap}} = \frac{d}{\mu_0 A}$$

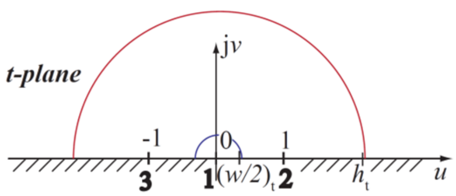
Derivation of new model

The Schwarz-Christoffel Transformation for Air Gaps



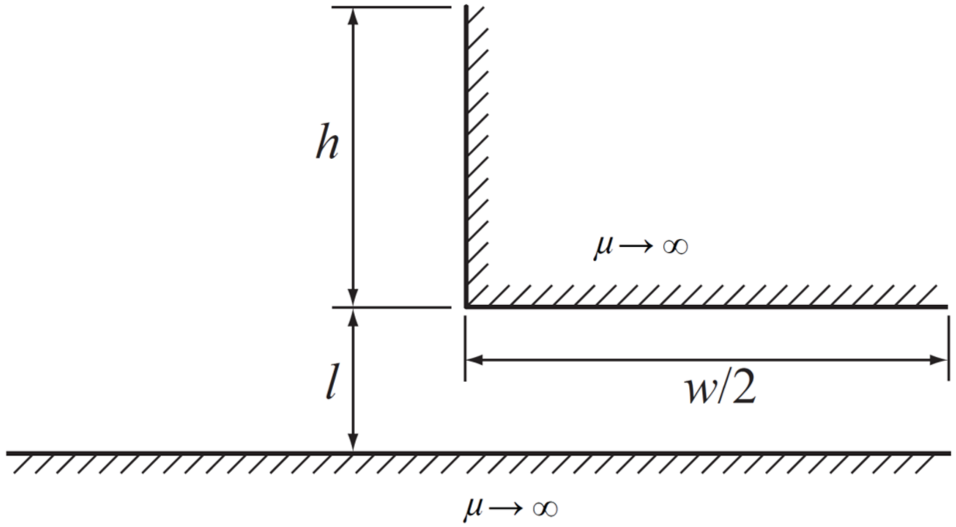


Schwarz-Christoffel Transformation



Derivation of new model

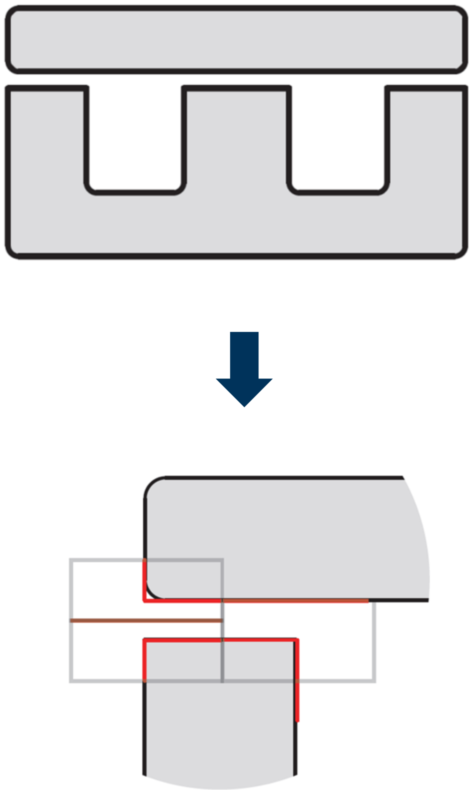
Basic Structure for the Air Gap Calculation (2-D)



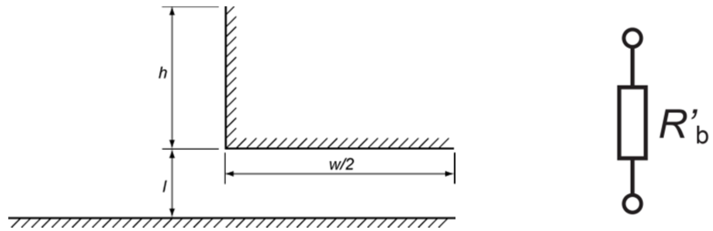
$$R'_{\text{basic}} = \frac{1}{\mu_0 \left[\frac{w}{2l} + \frac{2}{\pi} \left(1 + \ln \frac{\pi h}{4l} \right) \right]}$$

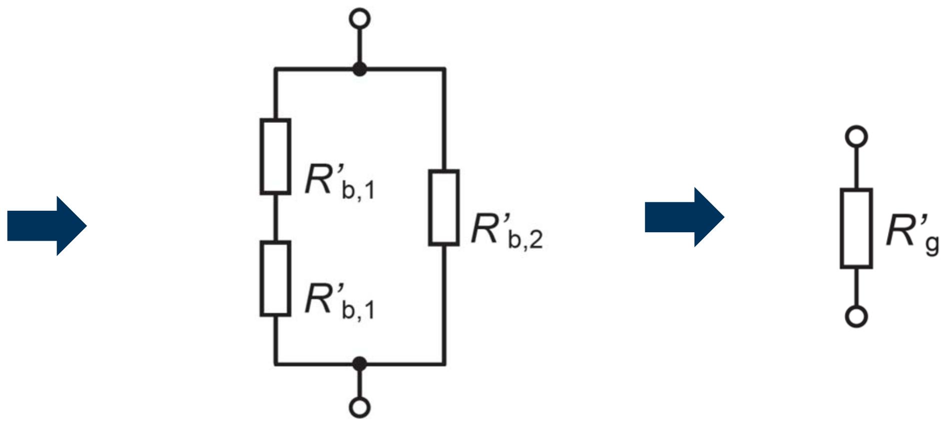
Derivation of new model

2-D (1)



Basic structure for the air gap calculation:

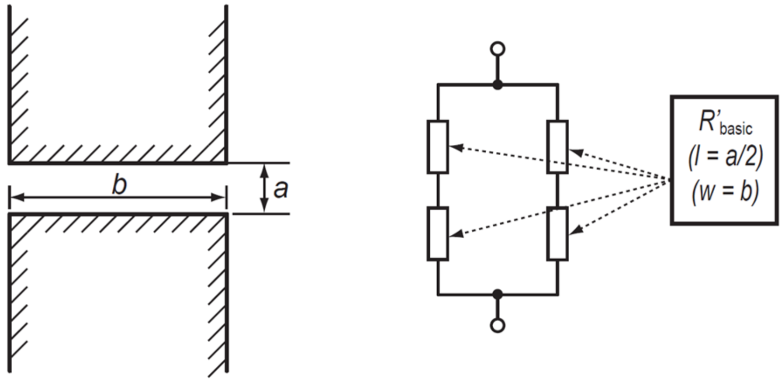




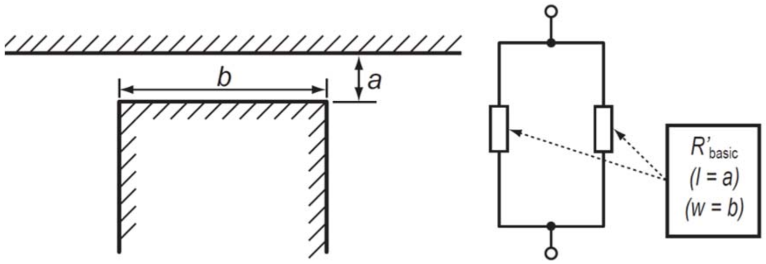
Derivation of new model

2-D (2)

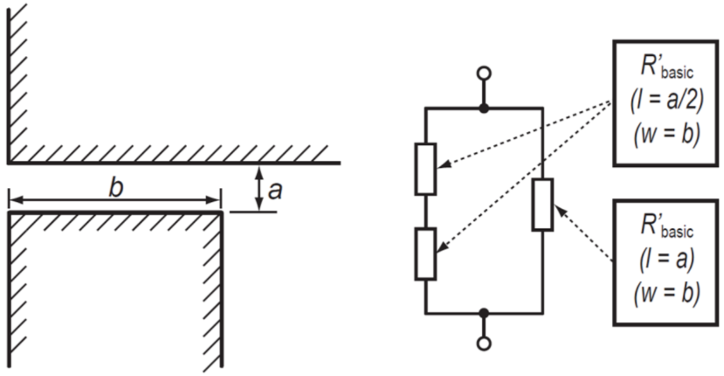
Air gap type 1



Air gap type 3



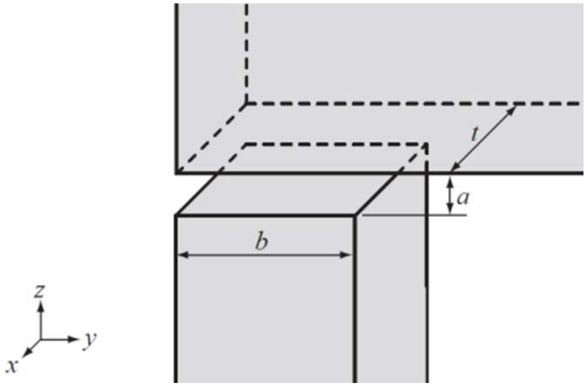
Air gap type 2



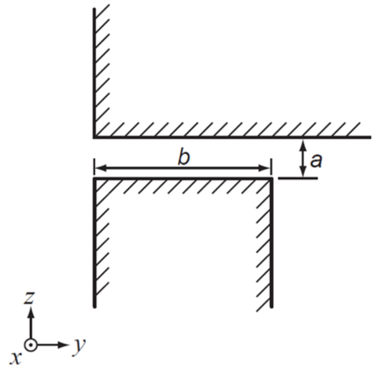
Derivation of new model

2D → 3D : Fringing Factor (1)

Illustrative Example

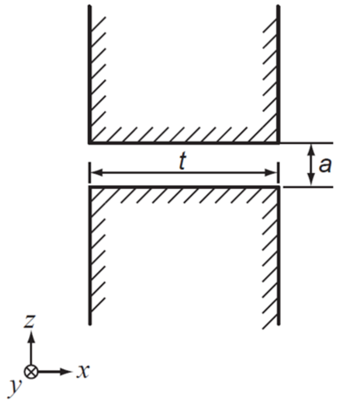


***zy*-plane**



➡ R'_{zy}

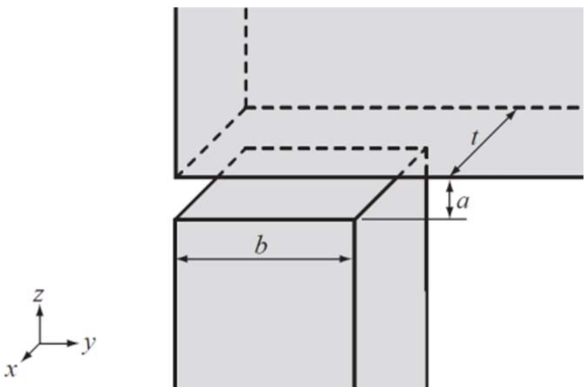
***zx*-plane**



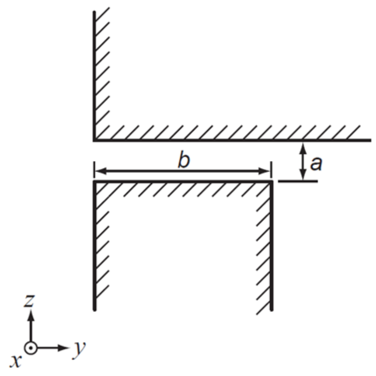
➡ R'_{zx}

Derivation of new model
2D → 3D : Fringing Factor (2)

Illustrative Example



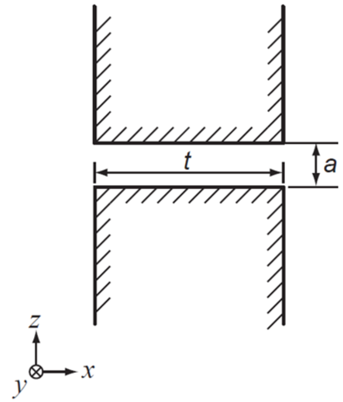
***zy*-plane**



$$\Rightarrow \quad \sigma_y = \frac{R'_{zy}}{\dfrac{a}{\mu_0 b}}$$

“Idealized” air gap (no fringing flux)

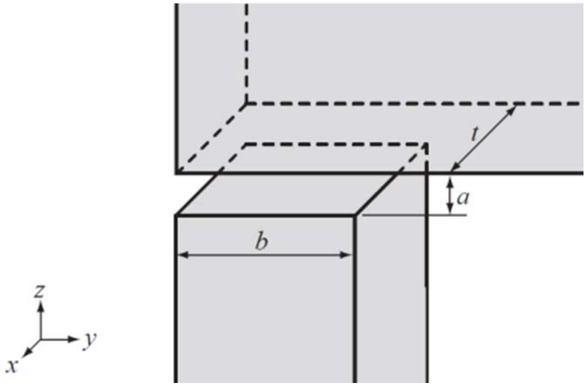
***zx*-plane**



$$\Rightarrow \quad \sigma_x = \frac{R'_{zx}}{\dfrac{a}{\mu_0 t}}$$

Derivation of new model
2D → 3D
Fringing Factor

Illustrative Example



3-D Fringing Factor: $$\sigma = \sigma_x \sigma_y$$

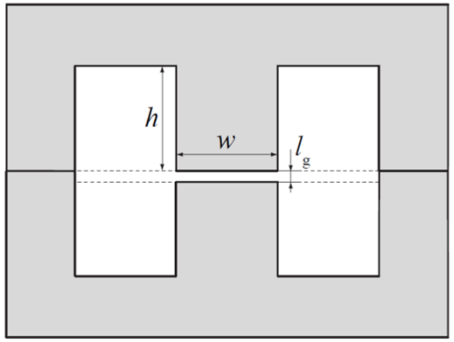
$$\Rightarrow \quad R_g = \sigma \frac{a}{\mu_0 b t}$$

"Idealized" air gap
(no fringing flux)

Results

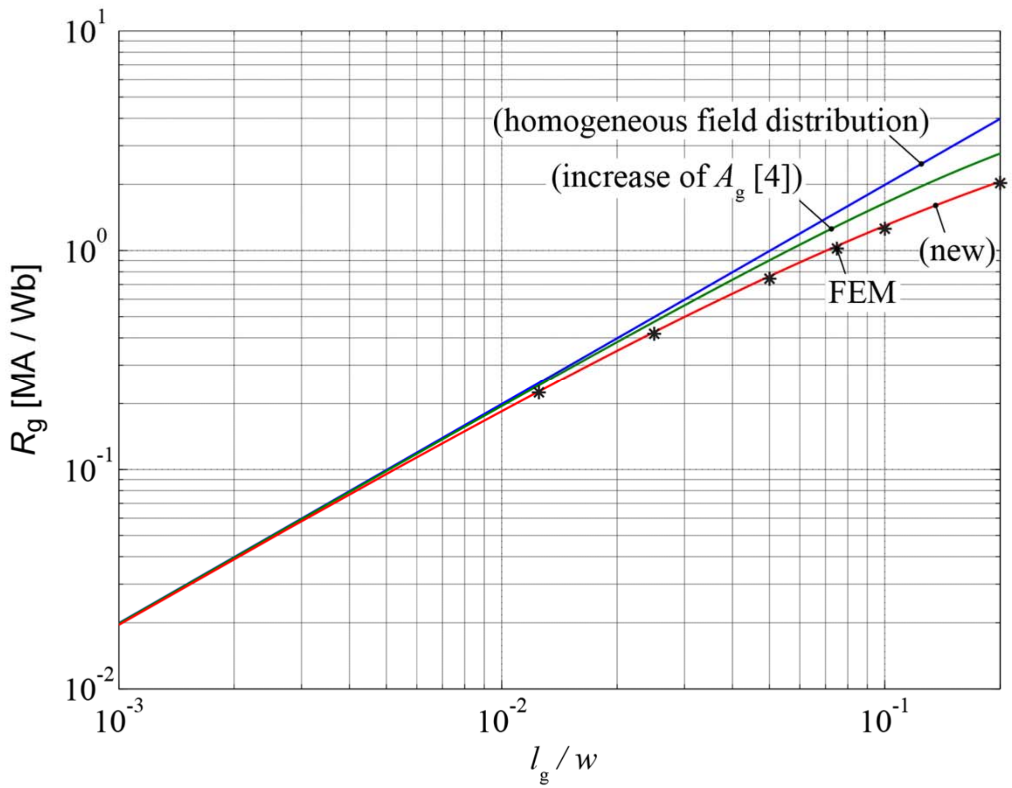
3-D FEM Simulation

Modeled Example



w = 40 mm; h = 40 mm

Results



Results
Experimental Results

Inductance Calculation

EPCOS E55/28/21, $N = 80$

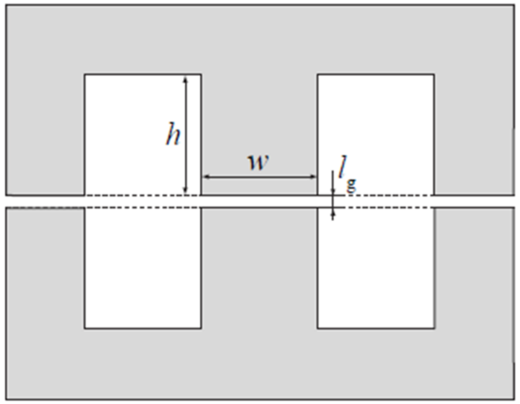


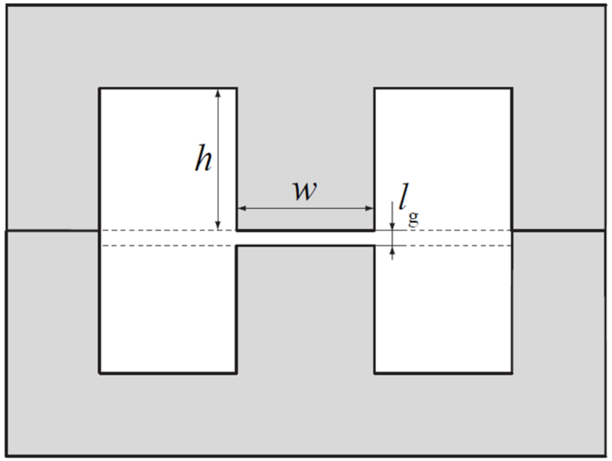
TABLE I
MEASUREMENT RESULTS OF E-CORE

Air Gap Length l_g	Calculated classically (3)	Calculated with new approach (12)	Measured
1.0 mm	1.42 mH	1.97 mH	2.07 mH
1.5 mm	0.96 mH	1.47 mH	1.58 mH
2.0 mm	0.72 mH	1.22 mH	1.26 mH

Results
Experimental Results

Saturation Calculation
EPCOS E55/28/21, N = 80,

l_g = 1 mm, B_{sat} = 0.45 T



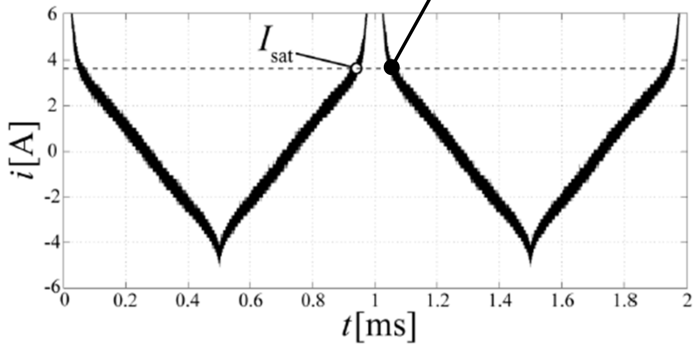
Calculations

TABLE II
MEASUREMENT RESULTS OF E-CORE

	Calculated classically (3)	Calculated with new approach (12)
L	2.75 mH	3.55 mH
I_{sat}	4.6 A	3.6 A

Measurement

I_{sat} = 3.7 A



Agenda

- Motivation
- Reluctance Model
- Core Losses
- Winding Losses
- Experimental Results

Core Losses

Different Modeling Approaches (1)

Steinmetz Equation SE

$$P = k\, f^{\alpha} B^{\beta}$$

- Only sinusoidal waveforms (→ iGSE).

iGSE

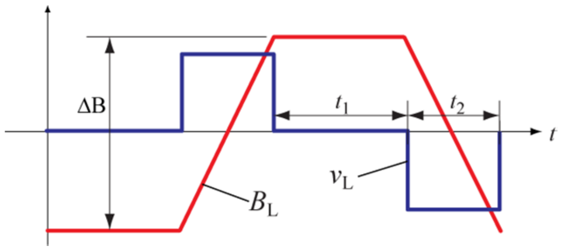
$$P_{\mathrm{v}} = \frac{1}{T}\int_{0}^{T} k_i \left|\frac{\mathrm{d}B}{\mathrm{d}t}\right|^{\alpha} (\Delta B)^{\beta-\alpha}\,\mathrm{d}t$$

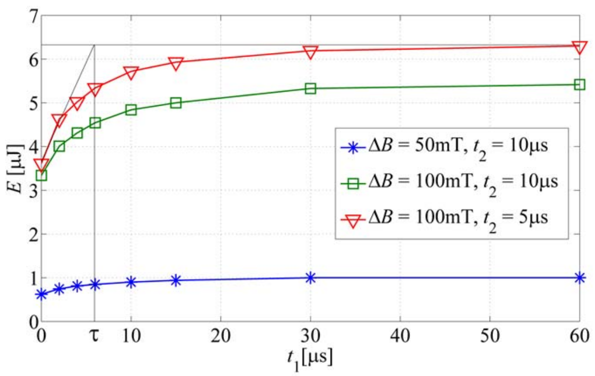
- DC bias not considered
- Relaxation effect not considered
- Steinmetz parameter are valid only in a limited flux density and frequency range

Core Losses

Different Modeling Approaches (2)

Relaxation Losses





Loss increase in the zero voltage phases (where dB/dt = 0)!

iGSE

$$P_v = \frac{1}{T}\int_0^T k_i \left|\frac{dB}{dt}\right|^{\alpha} (\Delta B)^{\beta-\alpha}\, dt$$

i²GSE

$$P_v = \frac{1}{T}\int_0^T k_i \left|\frac{dB}{dt}\right|^{\alpha} (\Delta B)^{\beta-\alpha}\, dt + \sum_{l=1}^{n} Q_{rl} P_{rl}$$

Core Losses
Different Modeling Approaches (3)

i²GSE

$$P_{\mathrm{v}} = \frac{1}{T}\int_0^T k_i \left|\frac{\mathrm{d}B}{\mathrm{d}t}\right|^{\alpha} (\Delta B)^{\beta-\alpha}\,\mathrm{d}t + \sum_{l=1}^{n} Q_{\mathrm{r}l} P_{\mathrm{r}l}$$

Remaining Problems

Steinmetz parameter are valid only in a limited flux density and frequency range.

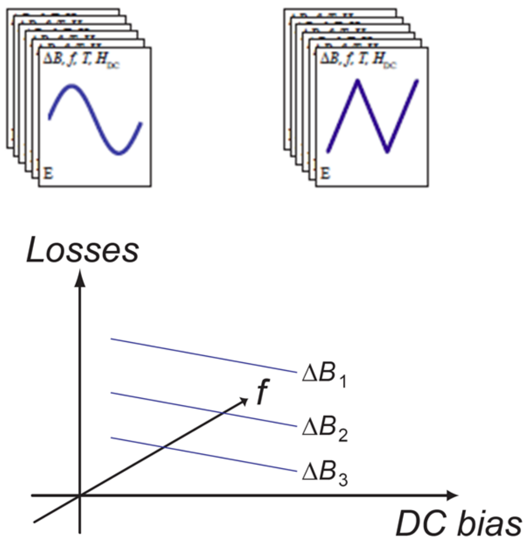
Core Losses vary under DC bias condition.

Modeling relaxation and DC bias effects need parameters that are not given by core material manufacturers.

➡ **Measuring core losses is indispensable!**

Core Losses
Different Modeling Approaches (4)

Loss Map
(Loss Material Database)

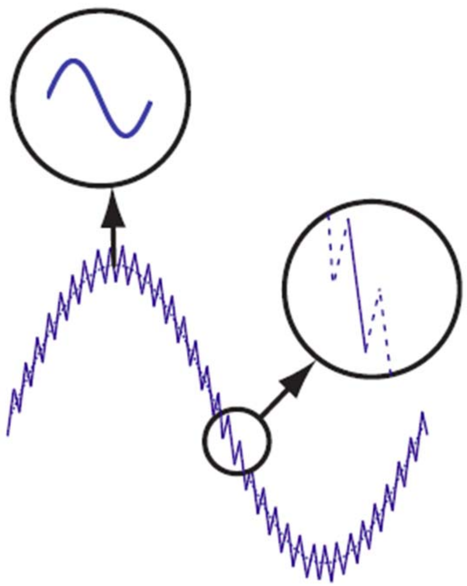


4th dimension: temperature.

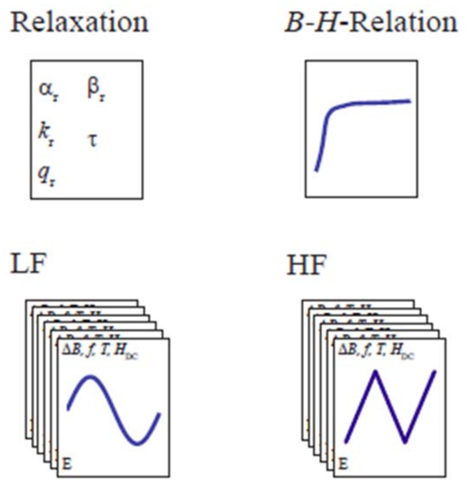
Question What loss map structure is needed to take all loss effects into consideration?

Core Losses
Needed Loss Map Structure

Typical flux waveform



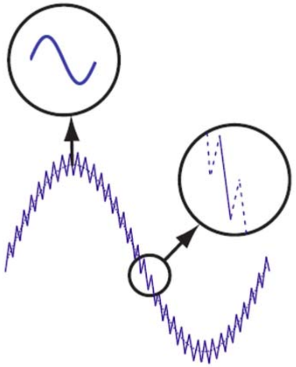
Content of Loss Map



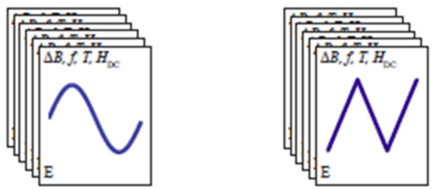
Core Losses

Hybrid Loss Modeling Approach (1)

“The best of both worlds”



Loss Map
(Loss Material Database)

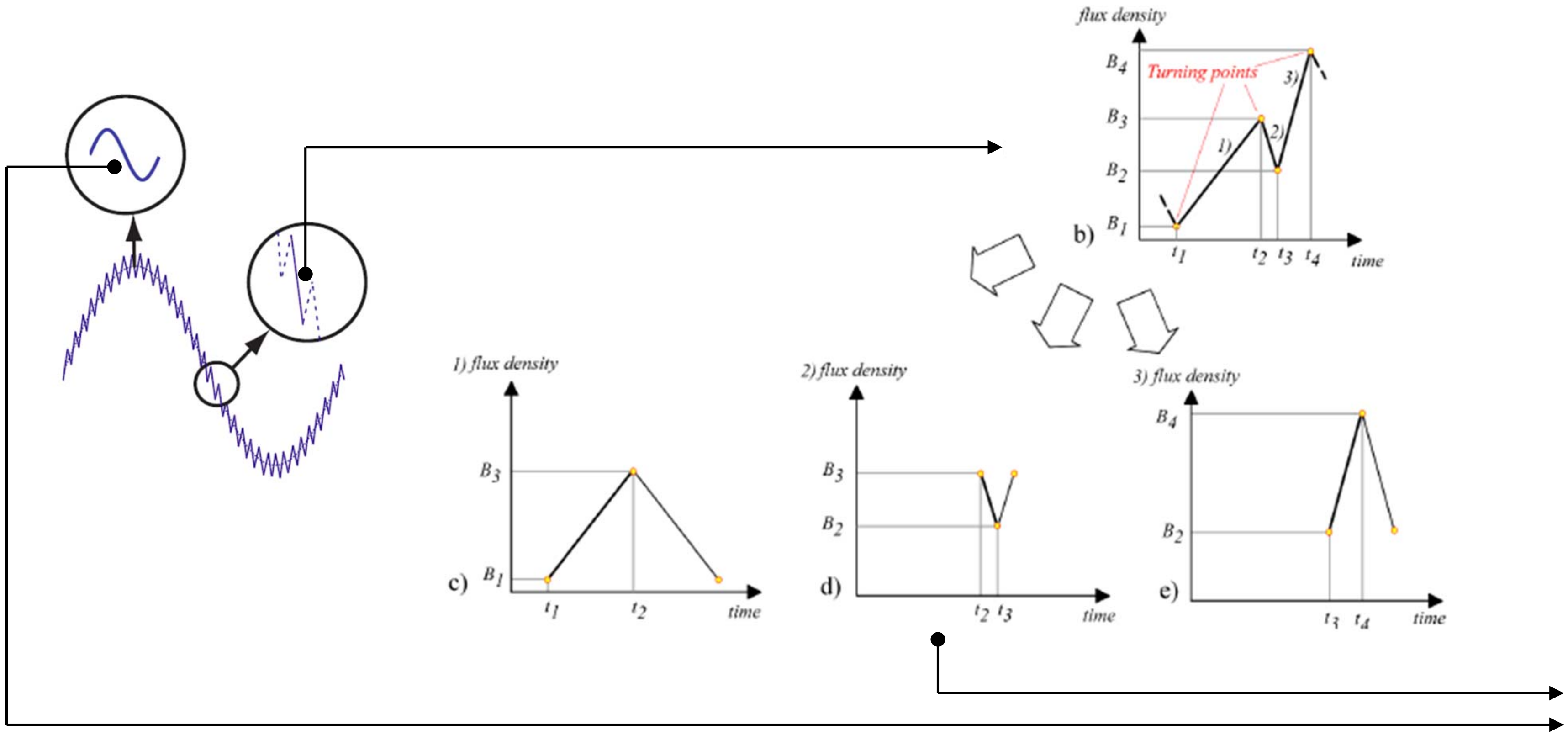


$$k, k_i, \alpha, \beta, \alpha_r, \beta_r, q_r, \tau$$

$$P_v = \frac{1}{T}\int_0^T k_i \left|\frac{dB}{dt}\right|^{\alpha} (\Delta B)^{\beta-\alpha}\, dt + \sum_{l=1}^{n} Q_{rl} P_{rl} \longrightarrow \frac{P}{V}$$

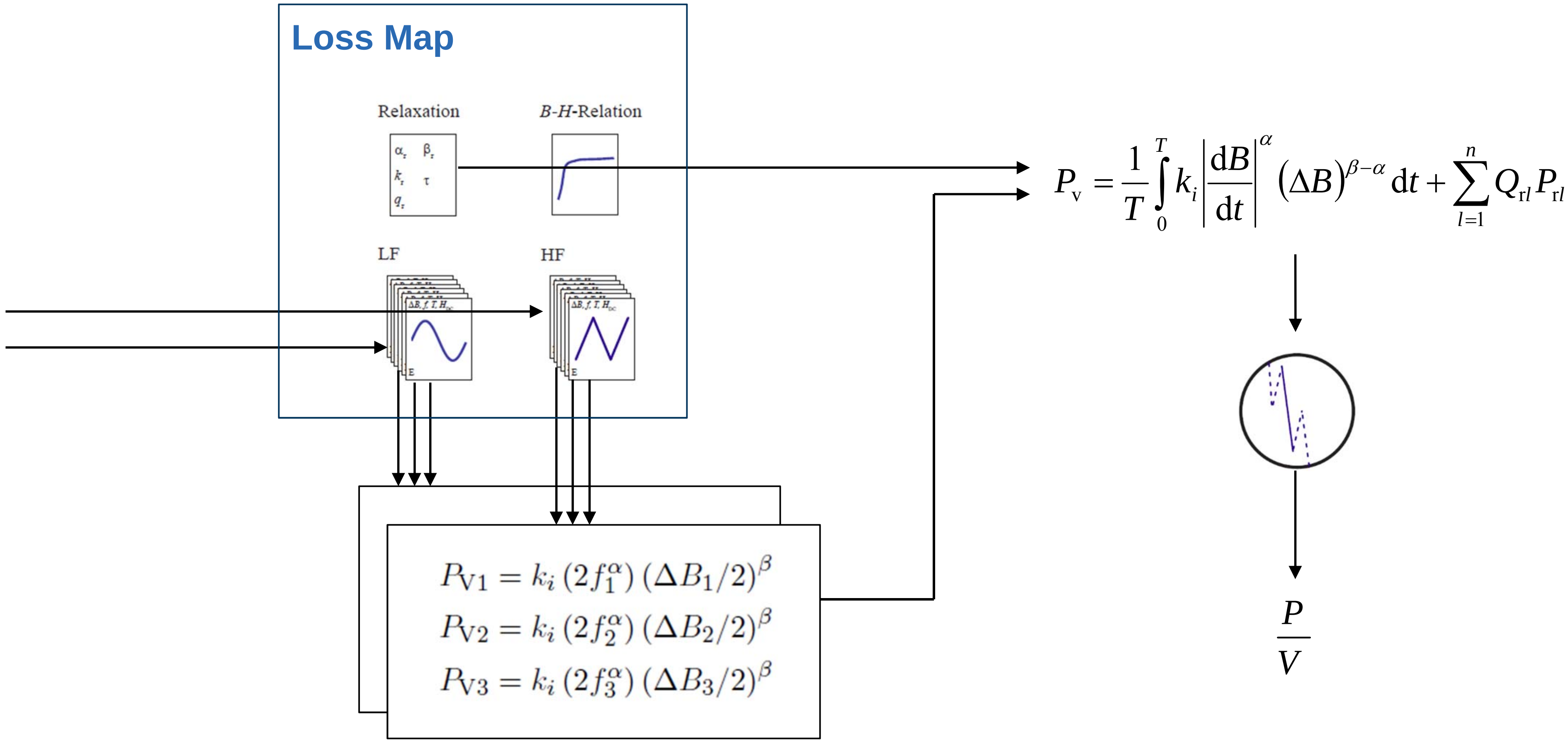
Core Losses

Hybrid Loss Modeling Approach (2)



Core Losses

Hybrid Loss Modeling Approach (3)



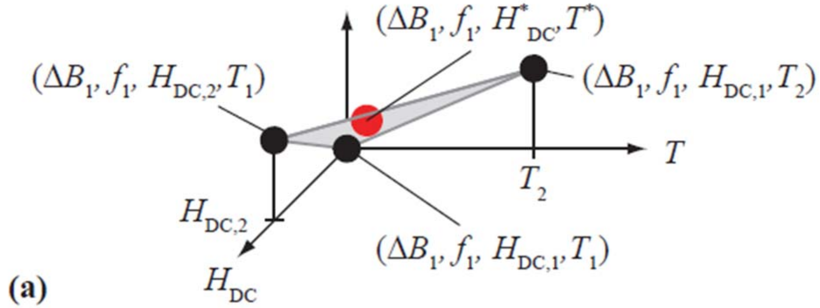
Core Losses

Hybrid Loss Modeling Approach (4)

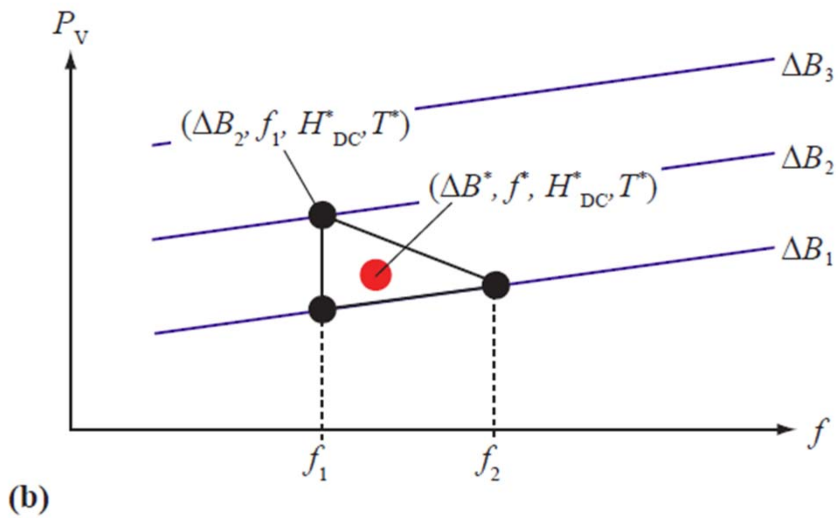
Interpolation and Extrapolation

(H_{DC}*, T*, ΔB*, f*)

H_{DC} and T



ΔB and f



Core Losses
Hybrid Loss Modeling Approach (5)

Advantages of combined approach (loss map and i²GSE):

- **Relaxation effects are considered (i²GSE).**
- **A good interpolation and extrapolation between premeasured operating points is achieved.**
- **Loss map provides accurate i²GSE parameters for a wide frequency and flux density range.**
- **A DC bias is considered as the loss map stores premeasured operating points at different DC bias levels.**

Core Losses
Effect of Core Shape

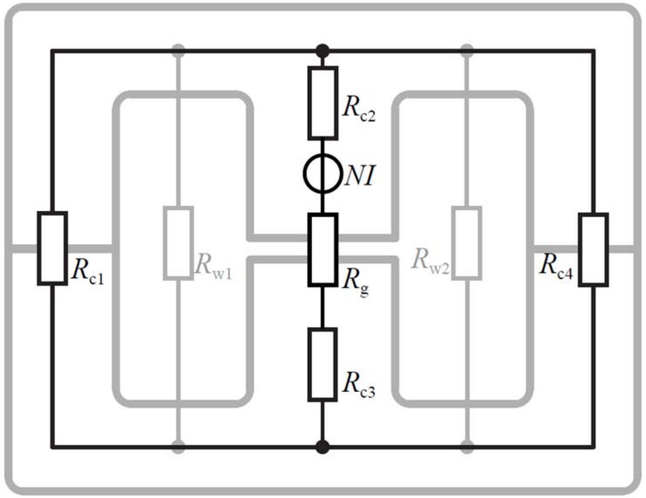
Procedure

1) The flux density in every core section of (approximately) homogenous flux density is calculated.

2) The losses of each section are calculated.

3) The core losses of each section are then summed-up to obtain the total core losses.

Reluctance Model



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Winding Losses
Skin Effect in Solid Round Conductors

$$P_{\mathrm{S}} = F_{\mathrm{R}}(f) \cdot R_{\mathrm{DC}} \cdot \hat{I}^2$$ (Loss per unit length)

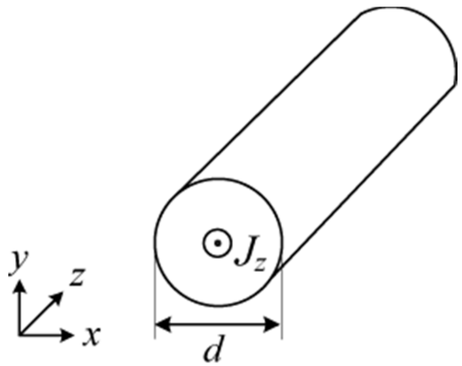
with

$$R_{\mathrm{DC}} = \frac{4}{\sigma \pi d^2}$$

$$\xi = \frac{d}{\sqrt{2}\delta}$$

$$\delta = \frac{1}{\sqrt{\pi \mu_0 \sigma f}}$$

$$F_{\mathrm{R}} = \frac{\xi}{4\sqrt{2}}\left[\frac{\mathrm{ber}_0(\xi)\mathrm{bei}_1(\xi) - \mathrm{ber}_0(\xi)\mathrm{ber}_1(\xi)}{\mathrm{ber}_1(\xi)^2 + \mathrm{bei}_1(\xi)^2} - \frac{\mathrm{bei}_0(\xi)\mathrm{ber}_1(\xi) - \mathrm{bei}_0(\xi)\mathrm{bei}_1(\xi)}{\mathrm{ber}_1(\xi)^2 + \mathrm{bei}_1(\xi)^2}\right]$$

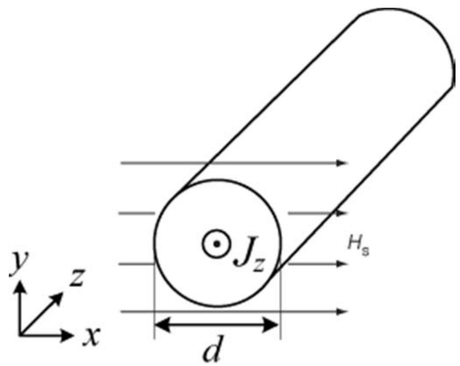


Winding Losses

Proximity Effect in Solid Round Conductors

$$P_{\mathrm{P}} = G_{\mathrm{R}}(f) \cdot R_{\mathrm{DC}} \cdot \hat{H}_{\mathrm{e}}^{2}$$ (Loss per unit length)

with



$$R_{\mathrm{DC}} = \frac{4}{\sigma \pi d^2}$$

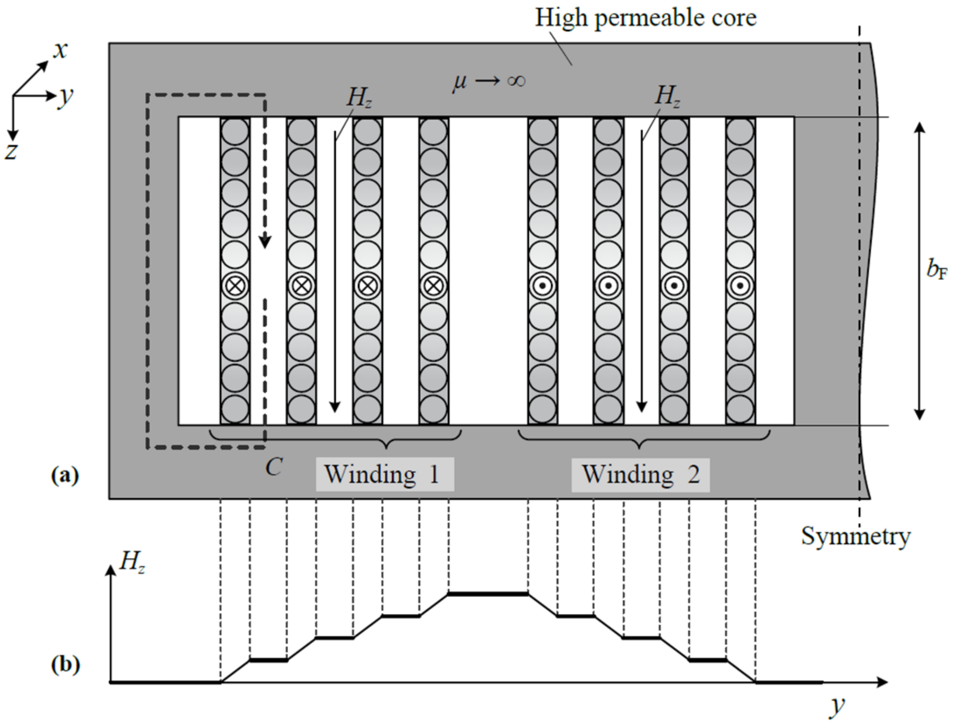
$$\xi = \frac{d}{\sqrt{2}\delta}$$

$$\delta = \frac{1}{\sqrt{\pi \mu_0 \sigma f}}$$

$$G_{\mathrm{R}} = -\frac{\xi \pi^2 d^2}{2\sqrt{2}} \left[\frac{\mathrm{ber}_2(\xi)\mathrm{ber}_1(\xi) + \mathrm{ber}_2(\xi)\mathrm{bei}_1(\xi)}{\mathrm{ber}_0(\xi)^2 + \mathrm{bei}_0(\xi)^2} + \frac{\mathrm{bei}_2(\xi)\mathrm{bei}_1(\xi) - \mathrm{bei}_2(\xi)\mathrm{ber}_1(\xi)}{\mathrm{ber}_0(\xi)^2 + \mathrm{bei}_0(\xi)^2} \right]$$

Winding Losses
Calculation of external field H_e (1D - approach)

Un-gapped cores (e.g. in transformers)



$$P = R_{DC}\left(F_R \hat{I}^2 NM + NG_R \sum_{m=1}^{M} \hat{H}^2_{avg,m}\right) l_m$$

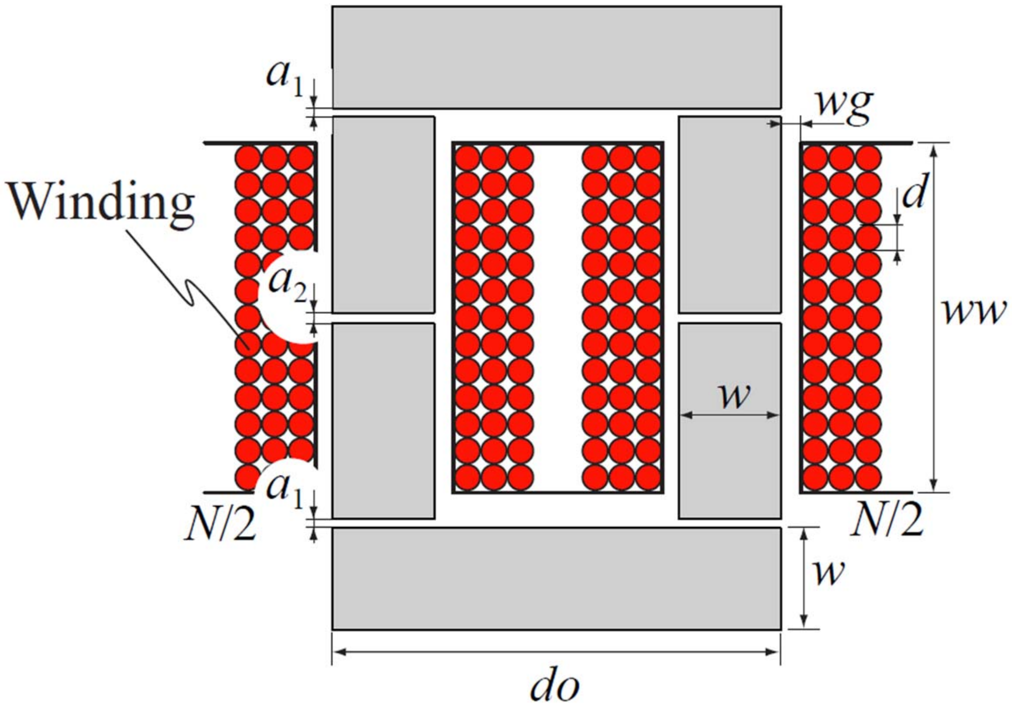
with

$$H_{avg} = \frac{1}{2}\left(H_{left} + H_{right}\right)$$

Winding Losses

Calculation of external field H_e (2D - approach)

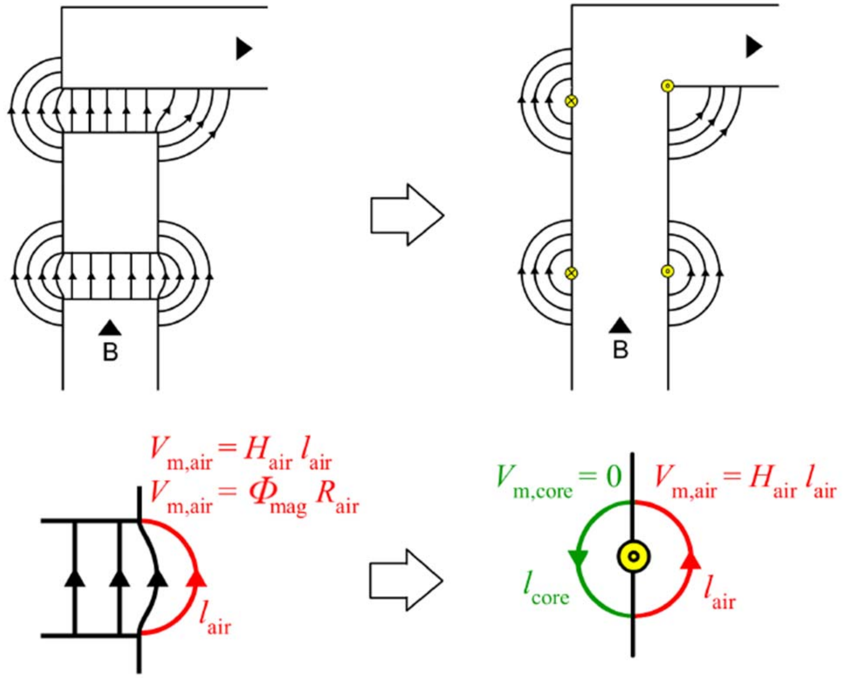
Gapped cores: 2D approach is necessary !



Winding Losses
Effect of the air gap fringing field

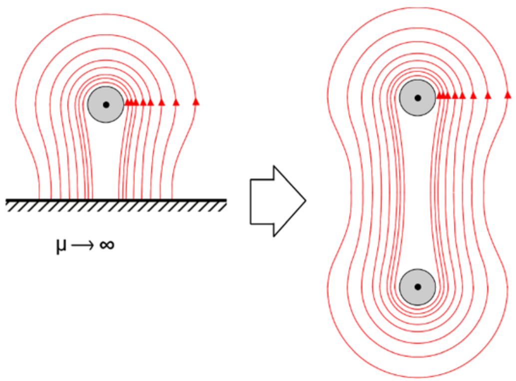
The air gap is replaced by a fictitious current, which …

… has the value equal to the magneto-motive force (mmf) across the air gap.

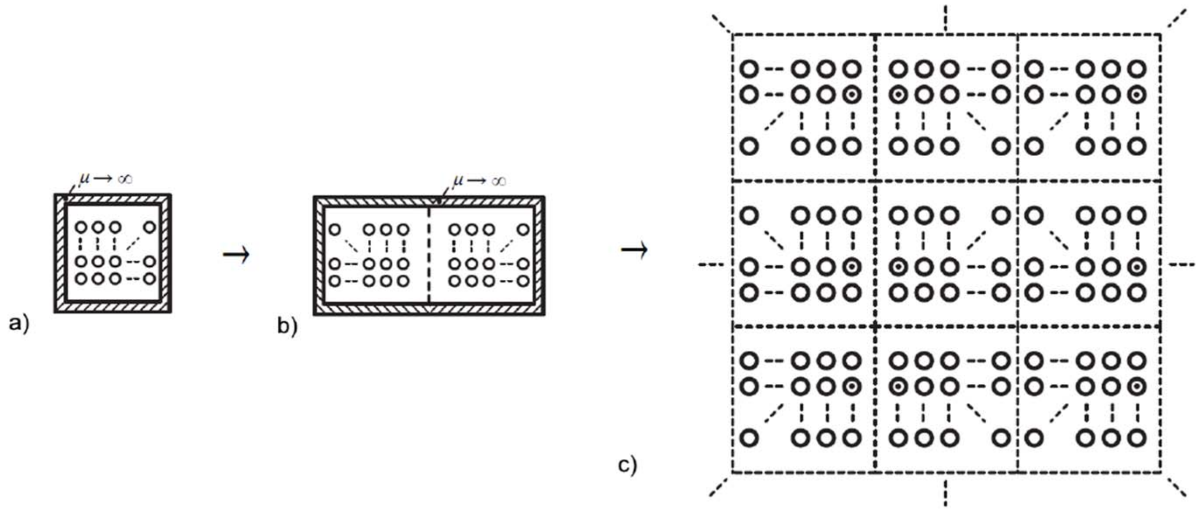


Winding Losses
Effect of the core material

The method of images (mirroring)

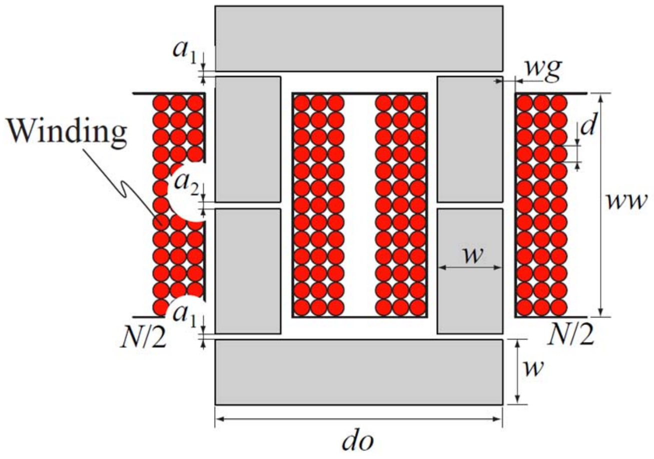


"Pushing the walls away"

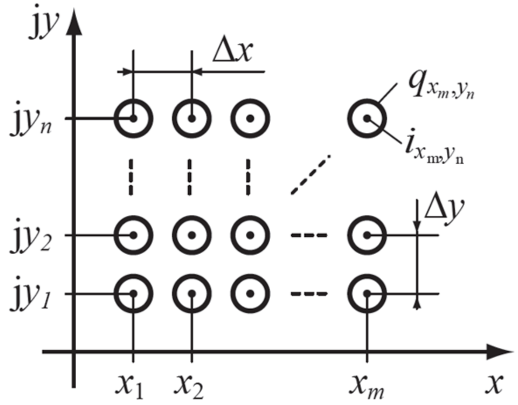


Winding Losses
Calculation of external field H_e (2D - approach)

Gapped cores: 2D approach



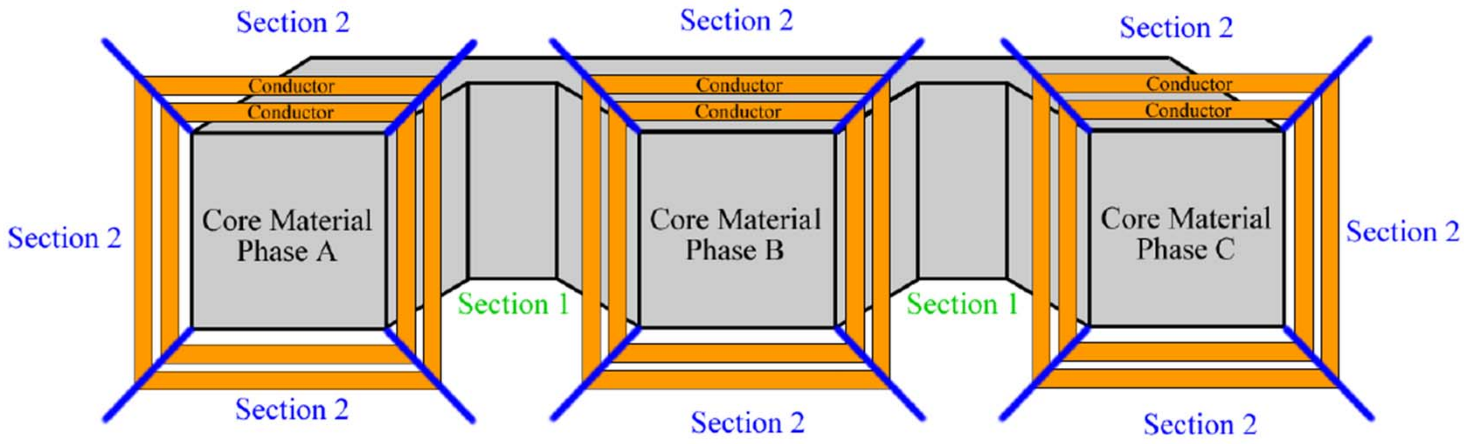
Winding Arrangement



External field vector across conductor $q_{xi;yk}$

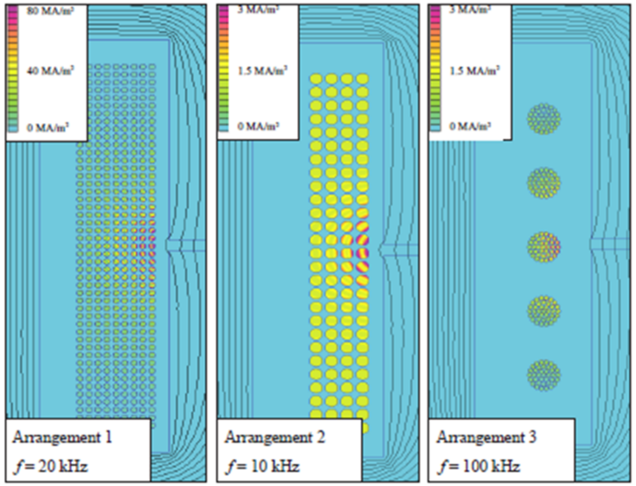
$$\hat{H}_e = \left| \sum_{u=1}^{m} \sum_{l=1}^{n} \epsilon(u,l) \frac{\hat{i}_{x_u,y_l} \left((y_l - y_k) - \mathrm{j}(x_u - x_i) \right)}{2\pi \left((x_u - x_i)^2 + (y_l - y_k)^2 \right)} \right|$$

Winding Losses
Different Winding Sections



Section 1: Core window - many mirrorings
Section 2: Not core window - one mirroring

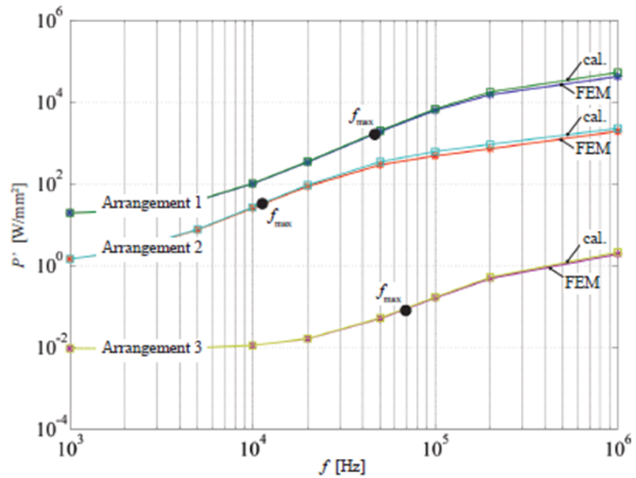
Winding Losses
FEM Simulations



Major Simplification

- magnetic field of the induced eddy currents neglected.
- This can be problematic at frequencies above (rule-of-thumb)

$$f_{\max} = \frac{2.56}{\pi \mu_0 \sigma d^2}$$



Results of considered winding arrangements

f-range	$f < f_{max}$	$f > f_{max}$
Error	< 5%	> 5% (always < 25%)

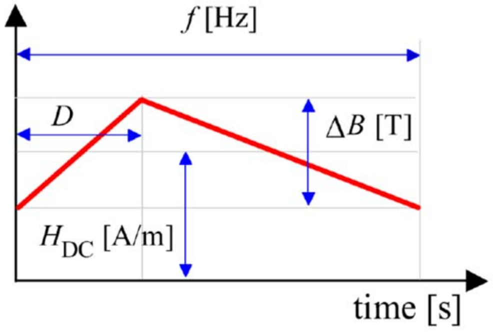
Agenda

- Motivation
- Reluctance Model
- Core Losses
- Winding Losses
- Experimental Results

Experimental Results

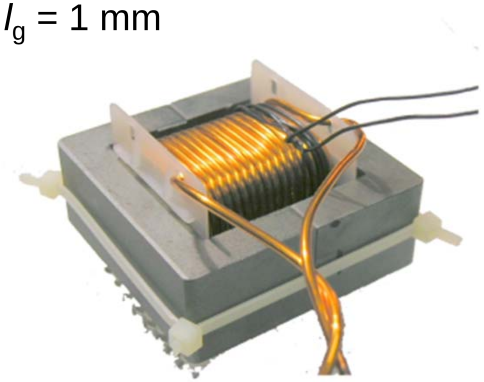
Measurement 1

Flux Waveform



Inductor

EPCOS E55/28/21, N = 18,
d = 1.7 mm, l_g = 1 mm

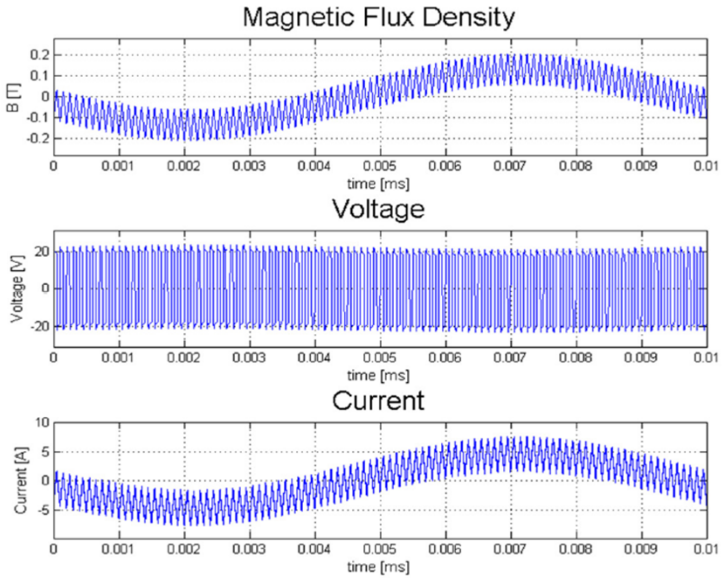


Results (measured with power analyzer Yokogawa WT3000)

Operating Points				Calculated Losses			Measured Losses	Comparison
ΔB [T]	f [kHz]	Duty-Cycle	H_{DC} [A/m]	Core Losses [W]	Winding Losses [W]	Total Losses [W]	Total Losses [W]	Rel. Error [%]
0.25	1	0.5	0	0.06	0.11	0.17	0.18	-5.56
0.5	1	0.5	0	0.32	0.42	0.74	0.77	-3.90
0.25	2	0.5	0	0.14	0.12	0.26	0.27	-3.70
0.5	2	0.5	0	0.72	0.49	1.21	1.21	0.00
0.25	5	0.5	0	0.4	0.23	0.63	0.61	3.28
0.5	5	0.5	0	2.04	0.92	2.96	2.70	9.63

Experimental Results
Measurement 2

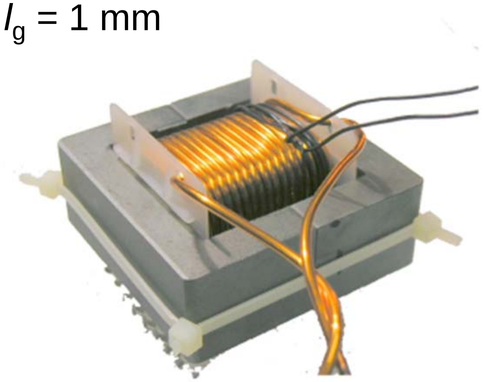
Flux Waveform



f_{LF} = 100 Hz
f_{HF} = 10 kHz

Inductor

EPCOS E55/28/21, N = 18,
d = 1.7 mm, l_g = 1 mm



Results (measured with power analyzer Yokogawa WT3000)

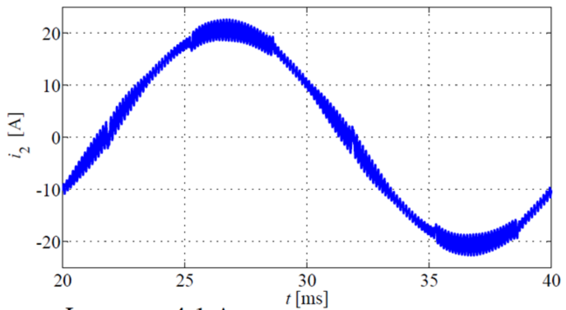
Operating Points			Calculated Losses			Measured Losses	Comparison
$\hat{I}_{LF}$ [A]	ΔB_{LF} [T]	ΔB_{HF} [T]	Core Losses [W]	Winding Losses [W]	Total Losses [W]	Total Losses [W]	Rel. Error [%]
5	0.25	0.15	0.44	0.35	0.79	0.76	3.95
10	0.5	0.3	1.83	1.51	3.35	3.6	-6.94

Experimental Results 3

Comparison Circuit Simulated Modeling vs. Measurements on a Boost Inductor Employed in a Three-Phase PFC Rectifiers [3]

Results

Simulated Current Waveform

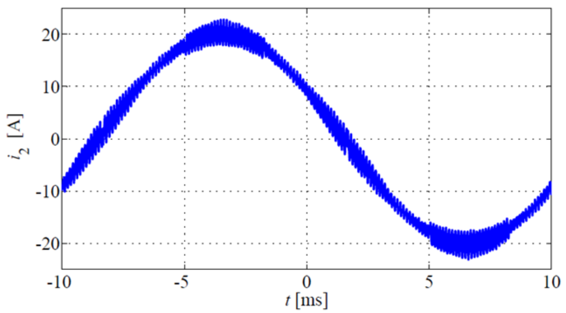


$I_{HF,pp,max} = 4.1$ A

$P_{Loss} = 43.3$ W

(a)

Measured Current Waveform



$I_{HF,pp,max} = 4.7$ A

$P_{Loss} = 46.8$ W

(b)

Photo

$L_2 = 2.54$ mH

Dimensions
(cf. Fig. 1)

N	94
a	1.95 mm
do	60 mm
h	60 mm
w	20 mm
t	28 mm
ww	56.9 mm
d	2.24 mm

Conclusion

Loss modeling very accurate.

Conclusion & Outlook

A high accuracy in modeling inductive components has been achieved.

The following effects have been considered:

Reluctance model [1]:

- Air gap stray field.
- Non-linearity of core material considered.

Core Losses [2]:

- DC Bias (Loss Map)
- Relaxation Effects (i^2GSE)
- Different flux waveforms (iGSE / i^2GSE)
- Wide range of flux density and frequencies (Loss Map)

Winding Losses [2]:

- Skin and proximity effect
- Stray field proximity effect
- Effect of core material

Thank you !

Do you have any questions ?

References

[1] J. Mühlethaler, J.W. Kolar, and A. Ecklebe, "A Novel Approach for 3D Air Gap Reluctance Calculations", in *Proc. of the ICPE - ECCE Asia*, Jeju, Korea, 2011

[2] J. Mühlethaler, J.W. Kolar, and A. Ecklebe, "Loss Modeling of Inductive Components Employed in Power Electronic Systems", in *Proc. of the ICPE - ECCE Asia*, Jeju, Korea, 2011

[3] J. Mühlethaler, M. Schweizer, R. Blattmann, J.W. Kolar, and A. Ecklebe, "Optimal Design of LCL Harmonic Filters for Three-Phase PFC Rectifiers", in *Proc. of the IECON, Melbourne,* 2011

[4] N. Mohan, T. M. Undeland, and W. P. Robbins - "Power Electronics – Converter, Applications, and Design", John Wiley & Sons, Inc., 2003