REALIZATION CONSIDERATIONS FOR UNIDIRECTIONAL THREE-PHASE PWM RECTIFIER SYSTEMS WITH LOW EFFECTS ON THE MAINS

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Abstract. The paper treats the analysis of power electronic circuits for unidirectional power conversion between a three-phase and a DC voltage system (e.g., the DC voltage link of an AC motor drive system or of an electronic welding current source). The application of turn-off power electronic devices on the DC voltage side or on the AC voltage side of a three-phase diode bridge makes possible the controllability of the DC link voltage such that it becomes independent of the mains voltage variations and of the applicable mains voltage level. (This becomes especially important for application with European, US, Japanese, etc. voltage systems.) There is offered the possibility of setting the DC voltage level to the highest possible value which guarantees a maximum utilization of the rated power of a connected converter unit. Furthermore there follows a considerable reduction of the effects on the mains as compared to a simple diode rectification.

Keywords: Unidirectional PWM Rectifier System, Control of the DC Link Voltage, Minimization of the Effects on the Mains, Three-Level Boost Converter

1 Introduction

The replacement of the classic line-commutated power electronic systems by self-commutated ones is discussed more and more frequently. This is mainly due to the substantial progress in semiconductor technology and to the related availability of turn-off power electronic devices. They allow the construction of pulse width modulated (PWM) converter systems in the lower and medium power region with high pulse rates; therefore they make possible current and voltage formation which can be freely given to a large extent. Here the background of these considerations is given by the possible reduction of the effects on the mains regarding harmonics, the improvement of the power factor and the possibility of full utilization of the rated power of a power electronic system to be supplied.

As an example we want to mention a circuit called PWM rectifier ([1], [2], [3], [4], [5]). This system (cf. Fig.1) appears to be ideally applicable to the supply of a DC link of an AC motor drive system from the three-phase mains. This is due to the fact that, besides controlling the DC link voltage, drawing a sinusoidal mains current with freely selectable phase angle is possible. This includes the possibility of feeding energy back to the mains.

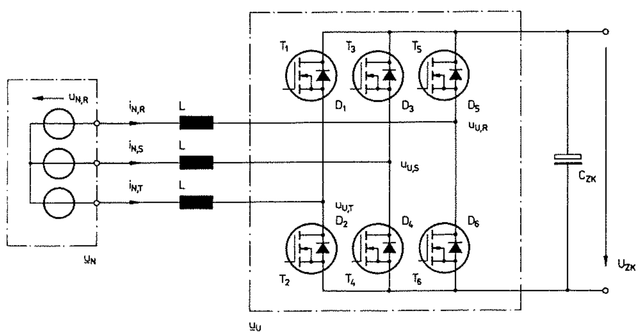


Fig.1: Structure of the power circuit of a bidirectional voltage DC link PWM rectifier system. (The voltage system $\underline{u}_N$ is defined by the mains conditions.)

For a decision concerning the broad industrial application of any system naturally besides technical aspects also economical points of view have to be considered. This brings up a basic drawback of the system described due to the relatively high number of power electronic and signal electronic devices necessary. With regard to a minimization of the system cost the question concerning possible modifications arises, combined with their influence on the system behavior. This question is the main topic of this paper. It is analyzed using unidirectional three-phase PWM rectifier systems. The restriction to only one energy flow direction is motivated by the then lower system cost. There does not follow an essential limitation of the system application in the lower power region.

The basic unit of the power electronic circuits discussed in the following is given by a three-phase diode bridge whose AC side extension by using turn-off power electronic devices leads to a class of self-commutated converters with DC voltage link or DC current link.

If turn-off power electronic devices are used on the DC side, there still line commutation is applied. The structure of the power circuit becomes simpler as compared to the self-commutated circuits mentioned. However, on the other hand there follow technical limitations given by the reduced control dynamics of the DC link voltage and the resulting effects on the mains.

2 Unidirectional Voltage DC Link PWM Rectifier System

As a closer analysis shows, the bridge legs the power electronic circuit shown in Fig.1 can be replaced (regarding their function) by three double-pole switches between positive and negative DC link voltage. The converter voltage space vector

$$\underline{u}_U = \frac{2}{3}\,(u_{U,R} + \underline{a}\;u_{U,S} + \underline{a}^2\;u_{U,T}) \qquad \underline{a} = \exp(j\frac{2\pi}{3}) \tag{1}$$

related to the switching state of the system takes on discrete directions (being separated by 60^0) in the complex plane. A given converter voltage reference vector

$$\underline{u}_U^* = \hat{\underline{U}}_U^*\exp(j\varphi_U) \qquad \varphi_U = \omega_N\tau \tag{2}$$

is approximated (as a time average over a pulse period) by the neighbouring converter voltage space vectors combined with the two not-voltage-forming switching states of the system. If only average values are considered (which are related to one pulse period) a converter output voltage system is formed which leads in connection with the mains voltage system

$$\underline{u}_N = \hat{\underline{U}}_N \exp(j\varphi_{\underline{u}_N}) \qquad \frac{d\varphi_{\underline{u}_N}}{dt} = \omega_N \tag{3}$$

to the current

$$\underline{i}_N^* = \hat{\underline{I}}_N \exp(j\varphi_{\underline{i}_N^*}) \qquad \varphi_{\underline{i}_N^*} = \omega_N \tau + \varphi_{\underline{u}_U^*, \underline{i}_N^*} \tag{4}$$

with the voltage difference

$$(\underline{u}_N - \underline{u}_U^*) = L \frac{d\underline{i}_N^*}{d\tau} \tag{5}$$

or

$$\underline{u}_U^*(\tau) = \underline{u}_N(\tau) - j\omega_N L\, \underline{i}_N^*(\tau)\ , \tag{6}$$

respectively. The phase angle (related to the mains voltage) and the magnitude of the input current then can be freely set in a large region by appropriate converter modulation and phase angle of the converter voltage space vector.

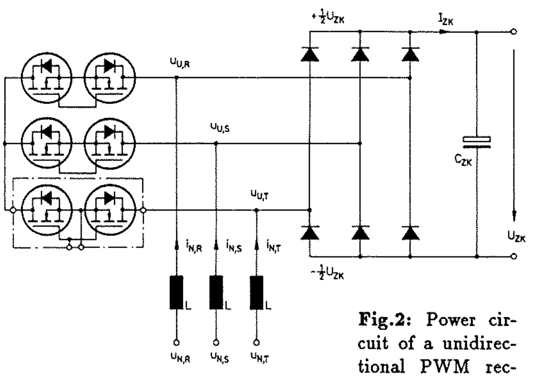
Dependent on the energy direction the current now flows into the DC link (where highly dynamic controllability of the DC link voltage is given) predominantly through the diodes (for energy supply to the load) or the transistors (for energy feedback into the mains) of the converter bridge legs. Regarding a limitation to only one energy flow direction one can now consider the replacement of the converter by a simple diode bridge. For a voltage generation which corresponds to the bidirectional system described (which finally becomes the condition for resulting of sinusoidal input currents and/or of an ideally constant power flow into the DC link) there have to be used turn-off power electronic devices on the AC side (cf. Fig.2). They have to be realized then, however, as bidirectional (current!) bipolar (blocking voltage!) power electronic devices due to the then resulting current flow directions and voltage polarities. The placement of the power electronic devices has to be performed such that converter voltage space vectors are formed by appropriate control of the line-to-line voltages whose combination leads to a voltage equilibrium with the mains voltage via the inductances connected in series. Besides the control signals of the power electronic devices there is also an influence given by the phase currents on the voltage formation. This is due to the fact that they define the resulting voltage conditions on the AC side of the rectifier bridge.

As a simple consideration shows, we have (if we assume that sinusoidal phase currents have to be generated) for the converter voltage state space vectors which have to be formed by the four possible system switching states

$$\varphi_{\underline{i}_N^*} \in [\frac{\pi}{6}, \frac{\pi}{2}] \quad \begin{array}{lcll} \underline{u}_{U,4} & = & \frac{2}{3} U_{ZK} & u_{U,ST} = 0 \\ \underline{u}_{U,6} & = & \frac{2}{3} U_{ZK} \exp(j\frac{\pi}{3}) & u_{U,RS} = 0 \\ \underline{u}_{U,2} & = & \frac{2}{3} U_{ZK} \exp(j\frac{2\pi}{3}) & u_{U,TR} = 0 \\ \underline{u}_{U,0} & = & \underline{u}_{U,7} = 0 & u_{U,RS} = u_{U,ST} = \\ & & & u_{U,TR} = 0\ . \end{array} \tag{7}$$

As becomes immediately clear from Fig.3 there will a system control be possible for practicable values of the short circuit voltages of the inductances connected in series by a cyclical change of the switching states such that the assumed sinusoidal phase currents can be formed. Because voltage space vectors can be formed only in a relatively close vicinity to the current space vector direction the mains current phase angle which can be set is restricted to small values. This does not result in a severe limitation due to the ohmic behavior relative to the mains as in most cases required for PWM rectifier systems.

A drawback of the circuit described is given by the relatively high conduction losses especially if the bidirectional bipolar valves are realized by only three controllable valves. A reduction of these losses is only possible by using the switch combination given in Fig.2. Due to the overall effort which is comparable to the bidirectional system this does not result in a technically or economically justifiable solution, however. Therefore, a more detailed discussion shall be omitted here.



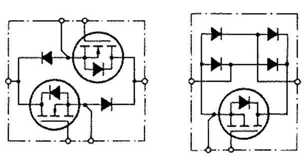
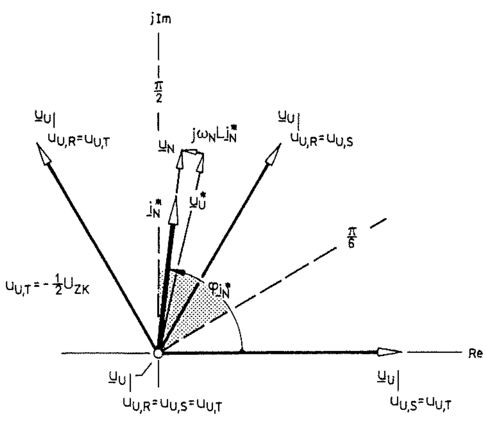


Fig.2: Power circuit of a unidirectional PWM rectifier system with DC voltage link. The placement of of the bidirectional bipolar power electronic devices can be made also as a Δ-connection.



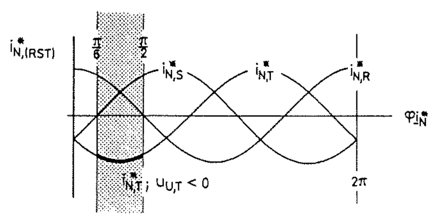


Fig.3: Voltage formation of a unidirectional PWM rectifier system with DC voltage link.

3 Unidirectional PWM Rectifier System with Current DC Link

As becomes immediately clear by considering the duality relationships ([6]) between voltage DC link and current DC link converters one can give a dual circuit for the circuit discussed in chapter 2 (cf. Fig.4 or [7], respectively). The function of this circuit can again be considered by using space vector calculus. A further analysis shall be omitted here for the sake of brevity.

4 Line Commutated Rectifier with Boost Converter on the DC Side

Besides an AC-side extension of a three-phase diode bridge by turn-off power electronic devices there is also a DC side placement of fully controllable valves thinkable. A variant is shown in Fig.5. The circuit shows an especially simple structure which is connected, however, with higher effects on the mains as compared to the circuits described before. The boost converter on the DC side defines the power flow into the DC link by controlling the DC link current. This current is mapped onto 120^0 mains current blocks according to the function of the line commutated diode

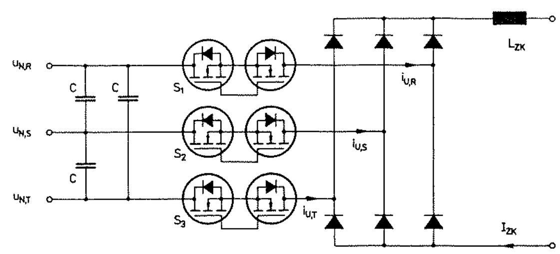


Fig.4: Power circuit of a unidirectional PWM rectifier system with DC current link.

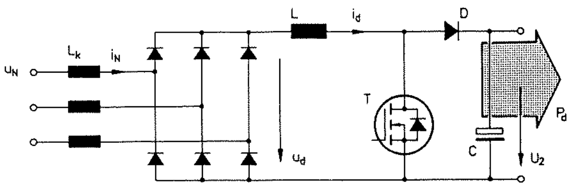


Fig.5: Line commutated rectifier with boost converter on the DC side.

bridge on the input side. The DC link voltage can then be controlled to a value above the line-to-line mains voltage peak value.

The resulting low-frequency mains harmonics show substantially reduced values as compared to simple diode rectification (the mains currents show pulse form (charging currents of the DC link capacitor), [8], [9]). They constitute a drawback, however, caused by the too low number of degrees of freedom of the control of the system (application of only one unidirectional unipolar turn-off power electronic device). Maintaining the principle of the simple system structure in the following we want to discuss the possibilities of reducing the effects on the mains.

A thought which suggests itself is given by smoothing the edges of the mains current blocks by connecting commutation inductances L_k in series (which might be given partially already by the mains inductances). Of main interest there is the dependency of the low frequency mains harmonics on the short circuit voltage u_k of the inductances or on the rated system power $P_{d,N}$. Using the derivation given in Appendix A there follows for the mains current shape (related to the peak value of the mains current fundamental $\hat{I}_{N,i,N}$ resulting for vanishing commutation inductances and rated power)

$$\begin{array}{ll} \varphi_N \in [-\frac{2\pi}{3}, (-\frac{2\pi}{3}+u_0)] & i'_N = \frac{\sqrt{3}}{2u'_k}\left[\sqrt{1-\frac{2\pi}{3}p'_d u'_k} - \cos(\omega_N t + \frac{2\pi}{3})\right] \\ \varphi_N \in [-\frac{\pi}{3}, (-\frac{\pi}{3}+u_0)] & i'_N = \frac{\sqrt{3}}{2u'_k}\left[1 - \cos(\omega_N t + \frac{\pi}{3})\right] \\ \varphi_N \in [(-\frac{\pi}{3}+u_0), +\frac{\pi}{3}] & i'_N = \frac{\sqrt{3}}{2u'_k}\left[1 - \sqrt{1-\frac{2\pi}{3}p'_d u'_k}\right] \\ \varphi_N \in [+\frac{\pi}{3}, (+\frac{\pi}{3}+u_0)] & i'_N = \frac{\sqrt{3}}{2u'_k}\left[\cos(\omega_N t - \frac{\pi}{3}) - \sqrt{1-\frac{2\pi}{3}p'_d u'_k}\right] \end{array} \tag{8}$$

with

$$0 \le u_0 \le \frac{\pi}{3} \tag{9}$$

or

$$p'_d \le \frac{9}{8\pi}\frac{1}{u'_k}\ , \tag{10}$$

respectively, and

$$i'_N(t) = \frac{1}{\hat{I}_{N,i,N}} i_N(t) \qquad t = \frac{\varphi_N}{\omega_N} \tag{11}$$

(cf. Fig.6). For the related DC link current we have

$$i'_d = \frac{3}{\pi}\frac{1}{u'_k}\left(1 - \sqrt{1 - \frac{2\pi}{3}p'_d u'_k}\right) \tag{12}$$

with

$$i'_d = \frac{I_d}{I_{d,i,N}} \qquad u'_k = \frac{U_{N,k}}{U_{N,N}}\ ; \tag{13}$$

the commutation overlap is given by

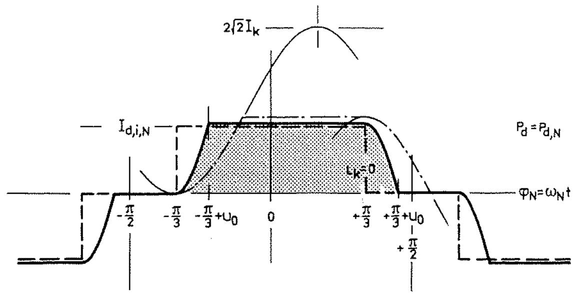


Fig.6: Calculation of the mains current shape or the mains current spectrum, respectively (cf. Fig.5).

$$\cos u_0 = \sqrt{1 - \frac{2\pi}{3}p'_d u'_k}\ . \tag{14}$$

The numerically calculated related mains current harmonics

$$\hat{i}'_{N,(\nu)} = \frac{\hat{I}_{N,(\nu)}}{\hat{I}_{N,i,N}} = c'_\nu = \sqrt{a'^2_\nu + b'^2_\nu} \tag{15}$$

with

$$\begin{aligned} a'_\nu &= \frac{2}{\pi}\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} i'_N(\omega_N t)\,\cos\nu\omega_N t\ d(\omega_N t) \\ b'_\nu &= \frac{2}{\pi}\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} i'_N(\omega_N t)\,\sin\nu\omega_N t\ d(\omega_N t) \end{aligned} \tag{16}$$

are shown in Fig.7. For a significant reduction of the low frequency mains current harmonics commutation inductances are necessary which are not reasonably realizable from a technical and economical point of view. High frequency mains current harmonics are reduced already for low values of the short circuit voltage according to the reduced steepness of the edges. This results in a reduced problem of the possibly given excitation of resonances in the mains by injection of high frequency current harmonics. In conclusion we have to state that the effects on the mains of the system can be influenced only marginally by connecting commutation inductances in series.

A further possibility of influencing the mains current spectrum is now thinkable by proper control of the DC link current. There by proper control of the boost converter a DC link current

$$i_d(t) = I_d + \sum_{\mu=6n} \hat{I}_{d,(\mu)}\cos\mu\omega_N t \qquad n = 1,2,3,\ldots \tag{17}$$

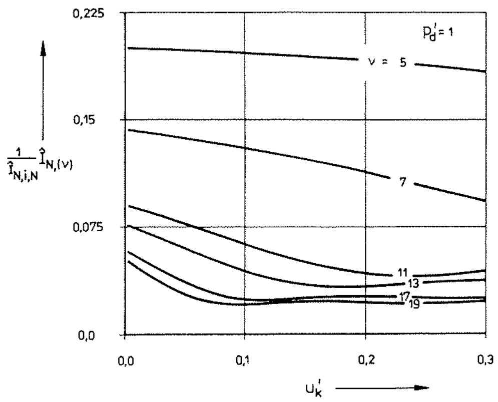


Fig.7: Mains current spectrum in dependency on the rated short circuit voltage u'_k of the inductances L_k connected in series on the AC side and on the rated output power p'_d (cf. Fig.5).

is injected, resulting in a mains current spectrum

$$i_N(t) = \sum_{\nu=1,5,7,\ldots} i_{N,(\nu)}(t) \qquad i_{N,(\nu)}(t) = \hat{I}_{N,(\nu)} \left\{ \sum_{\mu} \hat{I}_{d,(\mu)} \right\} \cos \nu\omega_N t \ . \tag{18}$$

The characteristic quantities for the possible influence on the mains current harmonics by the DC link current harmonics is given by the relative sensitivity

$$\epsilon_{\nu,\mu} = \frac{I_d}{\hat{I}_{N,(\nu),\mu\equiv 0}} \frac{d\hat{I}_{N,(\nu)}}{d\hat{I}_{d,(\mu)}} = \frac{\displaystyle\int_0^{\frac{\pi}{3}} \cos \nu\omega_N t \ \cos \mu\omega_N t \ d(\omega_N t)}{\displaystyle\int_0^{\frac{\pi}{3}} \cos \nu\omega_N t \ d(\omega_N t)} \ . \tag{19}$$

We have to note that (due to the limitation of the mains current blocks to a 120^0 interval given by the diode rectification) basically *all* mains current harmonics are influenced by a DC link current harmonic given by Eq.(17). An evaluation shows, however, (cf. Fig.8) that only such mains current harmonics are substantionally influenced which lie close to the DC link current harmonics (e.g., a DC link harmonic of order 6 influences primarily the harmonics 5 and 7 on the mains side of the system). The possibility of optimizing the mains current spectrum by injection of an appropriate DC link current harmonic is hardly given therefore. This is especially true because the influence on the mains current harmonics (side bands) due to a DC link current harmonic is in the opposite sense (cf. Fig.8). This means that the *reduction* of one harmonic on the mains results in an *increase* of the other harmonic which is primarily influenced.

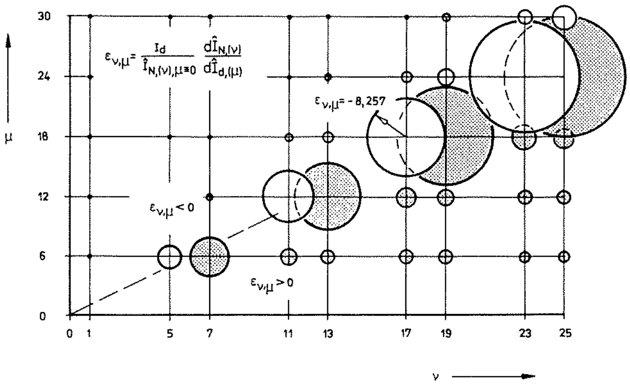


Fig.8: Possibility of influencing of the mains current harmonics by injection of DC link current harmonics shown by the relative sensitivity $\epsilon_{\nu,\mu}$.

The system given here therefore can be seen as the simplest variant of a three-phase supply unit which on one hand allows the control of the DC link voltage (the dynamics is limited to the line frequency, except for a load precontrol), which on the other hand shows, however, disadvantages concerning the resulting effects on the mains as compared to more complex systems.

5 Three-Level Boost Converter

In conclusion we want to briefly treat a possible variant of the realization of the boost converter situated in the DC link of the rectifier system described before.

Due to the requirement of a high DC link voltage (as mentioned already it has to be above the peak value of the line-to-line mains voltage) one has to use a series connection of capacitors. This is due to the limited dielectric strength of the electrolytic capacitors. As the following considerations show one can now extend the boost converter to a three-level boost converter (cf. Fig.9) by inclusion of the now resulting capacitive center point into the system function. This also leads to halving the blocking voltage on the valves and to a reduction of the rated power of the DC side inductance. Alternatively, the switching frequency can be reduced for equal inductance. A possibly required control of the voltage level of the capacitive center point (symmetrization of the capacitor voltages) can be performed by proper clocking of the two partial systems of the boost converter.

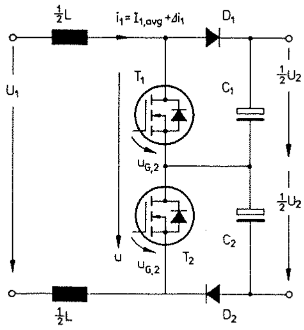


Fig.9: Power circuit of a three-level boost converter.

According to Fig.10 we have for the relationship between system input and output voltages (independent of γ, cf. Fig.11)

$$U_1 = (1-\delta) \ U_2 \ . \tag{20}$$

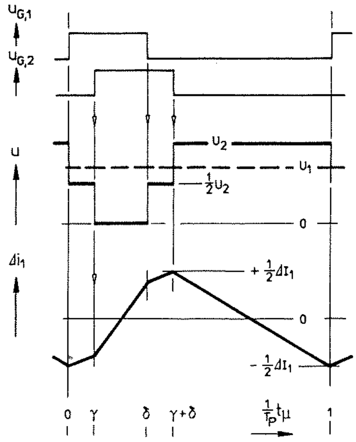


Fig.10: Control signals $u_{G,1}$ and $u_{G,2}$, inductor AC current part $\Delta i_1(t_\mu)$ and voltage u of a three-level boost converter ($0 \leq \delta \leq 0.5$, $0 \leq \gamma \leq 0.5$; $\delta = 0.3$, $\gamma = 0.1$).

The three possible voltage levels ("three-level" boost converter)

$$u = 0 \qquad u = \frac{U_2}{2} \qquad u = U_2 \tag{21}$$

which can be given by proper control lead according to

$$u_L = U_1 - u = L\frac{di_1}{dt} \tag{22}$$

to the ripple of the inductor current

$$\begin{array}{llrcl}
0 \leq \delta \leq 0.5 & 0 \leq \gamma \leq 0.5 & \Delta i_1' & = & \delta(1-\delta-\gamma) \\
0 \leq \delta \leq 0.5 & 1 \geq \gamma \geq 0.5 & \Delta i_1' & = & \delta(\gamma-\delta) \\
1 \geq \delta \geq 0.5 & 0 \leq \gamma \leq 0.5 & \Delta i_1' & = & (1-\delta)(\delta-\gamma) \\
1 \geq \delta \geq 0.5 & 1 \geq \gamma \geq 0.5 & \Delta i_1' & = & (1-\delta)(\delta+\gamma-1)
\end{array} \tag{23}$$

with

$$\Delta i_1' = \frac{1}{\Delta i_n}\Delta I_1 \qquad \Delta i_n = \frac{U_2 T_P}{L} \tag{24}$$

(cf. Fig.11). A minimization of $\Delta i_1'$ therefore is given for sequential clocking of the two converter parts according to

$$\gamma = 0.5 \ . \tag{25}$$

A comparison to the values of the simple boost converter ($\gamma = 0; 1$)

$$\frac{\Delta i'_{1,max}\Big|_{\gamma=0.5}}{\Delta i'_{1,max}\Big|_{\gamma=0;1}} = \frac{1}{4} \qquad \Delta i'_{1,max}\Big|_{\gamma=0.5} = \frac{1}{4} \tag{26}$$

shows a significant reduction of the inductor current ripple for the three-level boost converter (for equal inductance and switching frequency).

For the rms value of the current ripple there follows according to the relationships given in Appendix B (the considerations can be limited to the case given in the following, due to the symmetries relative to the axes $\delta = 0.5$ and $\gamma = 0.5$ which are clearly visible in Fig.11)

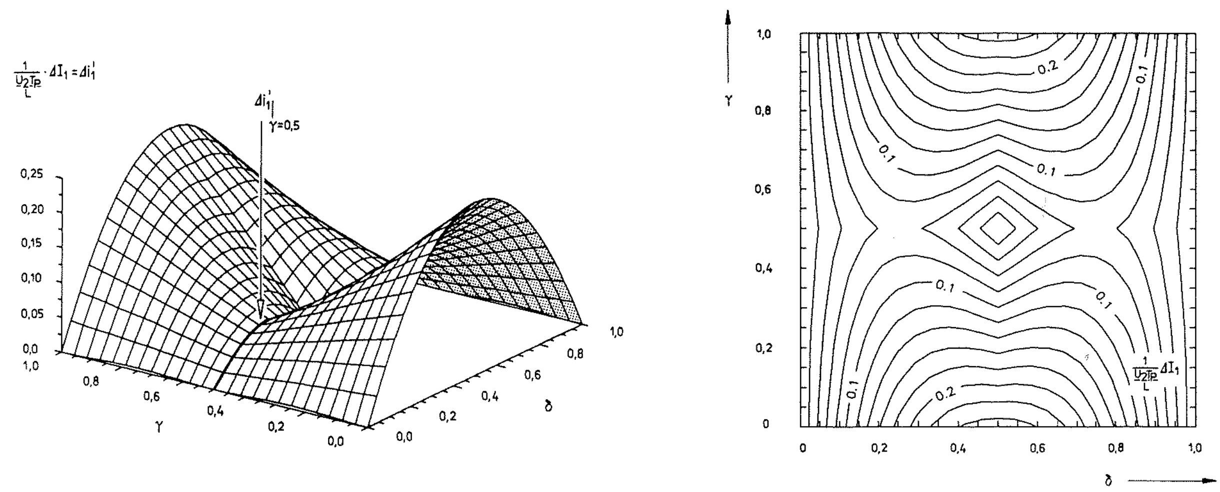


Fig.11: Dependency of the rated input current ripple $\Delta i_1'$ on the duty ratio δ (input voltage) and phase displacement γ of the clocking of the partial systems.

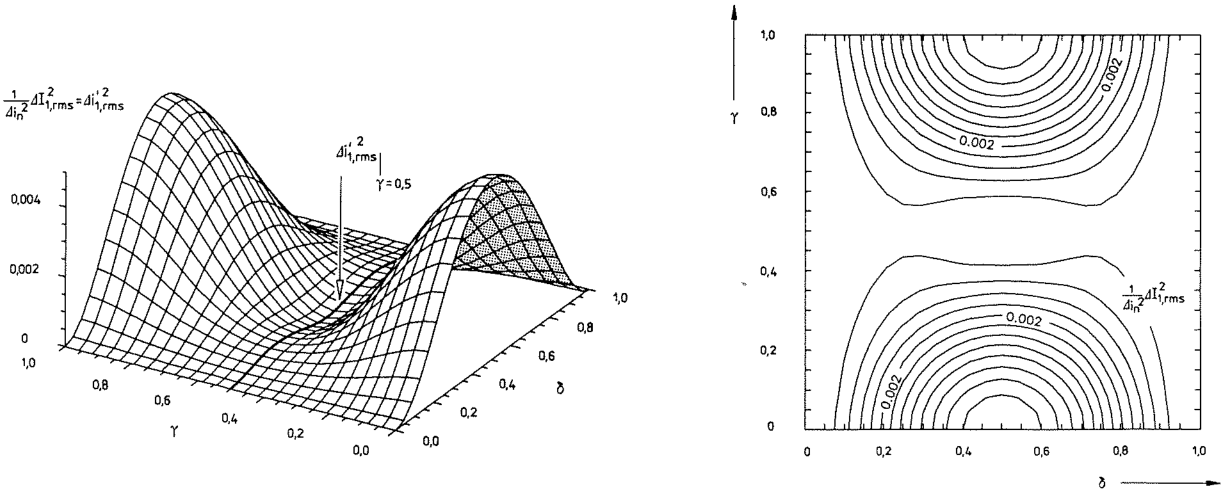


Fig.12: Rated square of the rms value of the inductor AC current part $\Delta i_{1,rms}'^2$ in dependency on the duty ratio δ and phase displacement γ of the clocking of the partial systems.

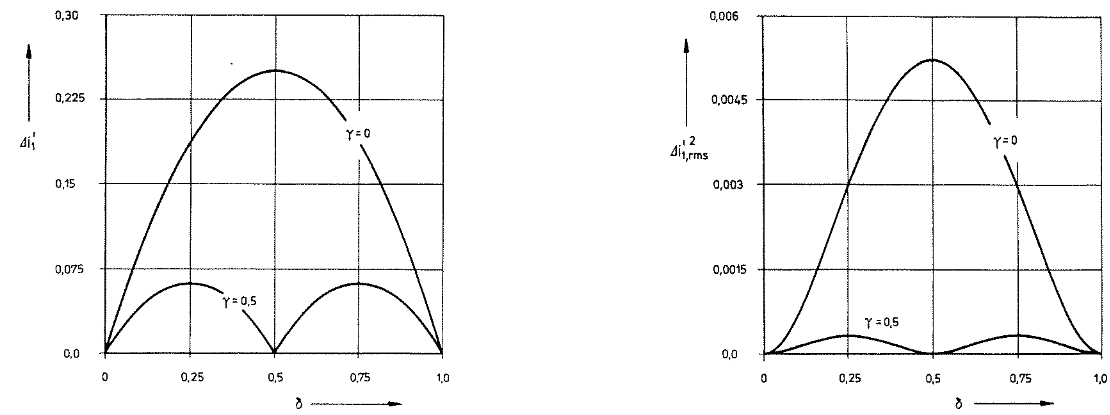


Fig.13: Peak-to-peak value $\Delta i_1'$ and square of the rms value of the rated inductor AC current part $\Delta i_{1,rms}'^2$ for opposite-phase clocking ($\gamma = 0.5$) of the partial systems.

$$0 \le \delta \le 0.5 \quad 0 \le \gamma \le \delta \qquad \Delta i_{1,rms}'^2 = \frac{1}{12}[\delta^4 - 2\delta^3 + \delta^2(3\gamma^2 + 1) - 3\delta\gamma^2 + \gamma^3]$$

$$0 \le \delta \le 0.5 \quad \delta \le \gamma \le 0.5 \qquad \Delta i_{1,rms}'^2 = \frac{\delta^2}{12}(\delta^2 - \delta + 3\gamma^2 - 3\gamma + 1) \tag{27}$$

with

$$\Delta i'^2_{1,rms} = \frac{1}{\Delta i_n^2} \Delta I^2_{1,rms} \tag{28}$$

(cf. Figs.12,13). A comparison with the simple boost converter leads with

$$\frac{\Delta i'^2_{1,rms,max}\Big|_{\gamma=0.5}}{\Delta i'^2_{1,rms,max}\Big|_{\gamma=0;1}} = \frac{1}{16} \qquad \Delta i'^2_{1,rms,max}\Big|_{\gamma=0.5} = \frac{1}{12}\frac{1}{16^2} \tag{29}$$

to the expected reduction of the harmonic losses to 1/16 if Eq.(26) is considered.

Finally, we have to state that the current stress on the DC link capacitors is not changed as compared to the simple boost converter.

6 Conclusions

In general, the applicability of the systems described in this paper is essentially determined by the cost of the applied semiconductor device combinations. These in turn are determined essentially by the application area and the number of devices used. This might result in the fact that a combination of semiconductor devices combined in a single package and forming a bridge leg might come close to the cost of a single device used in such a circuit when discrete devices are used. This fact always has to be considered (besides the complexity of the power and control circuits) for estimating the effort and cost of realization of such systems (as described, e.g., in this paper).

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Appendix A

$$U_{d,i} = \frac{3\sqrt{2}\sqrt{3}U_N}{\pi} \tag{A.1}$$

$$I_k = \frac{\sqrt{3}U_N}{2\omega_N L_k} \tag{A.2}$$

$$\cos u_0 = 1 - \frac{I_d}{\sqrt{2}I_k} \tag{A.3}$$

$$P_d = I_d U_d = I_d U_{d,i}\left(1 - \frac{I_d}{2\sqrt{2}I_k}\right) \tag{A.4}$$

$$\cos u_0 = 1 - \frac{2\omega_N L_k I_d}{\sqrt{2}\sqrt{3}U_N} \tag{A.5}$$

$$P_d = 3U_N I_d \frac{\sqrt{2}\sqrt{3}}{\pi}\left(1 - \frac{\omega_N L_k I_d}{\sqrt{2}\sqrt{3}U_N}\right) \tag{A.6}$$

$$I_{N,(1)}\Big|_{L_k=0;P_{d,N}} = I_{N,i,N} = \frac{\sqrt{2}\sqrt{3}I_{d,i,N}}{\pi} \tag{A.7}$$

$$u'_k = \frac{U_{N,k}}{U_{N,N}} = \frac{I_{N,i,N}\,\omega_N L_k}{U_{N,N}} = \frac{\sqrt{2}\sqrt{3}}{\pi}\frac{I_{d,i,N}\,\omega_N L_k}{U_{N,N}} \tag{A.8}$$

$$P_{d,N} = 3\,U_{N,N}\,I_{N,i,N} \tag{A.9}$$

$$i'_d = \frac{I_d}{I_{d,i,N}} \tag{A.10}$$

$$p'_d = \frac{P_d}{P_{d,N}} \tag{A.11}$$

$$p'_d = i'_d\left(1 - \frac{\pi}{6}i'_d u'_k\right) \tag{A.12}$$

$$\cos u_0 = \left(1 - \frac{\pi}{3}i'_d u'_k\right) \tag{A.13}$$

$$\begin{aligned} u_0 &= \tfrac{\pi}{6} \qquad & p'_d &= \tfrac{3}{8\pi}\tfrac{1}{u'_k} \\ u_0 &= \tfrac{\pi}{3} \qquad & p'_d &= \tfrac{9}{8\pi}\tfrac{1}{u'_k} \end{aligned} \tag{A.14}$$

Appendix B

$$I^2_{1,rms} = \frac{1}{T_P}\int_{t_\mu=0}^{t_\mu=T_P} i_1^2(t_\mu)\,dt_\mu \tag{B.1}$$

$$i_1(t_\mu) = I_{1,avg} + \Delta i_1(t_\mu) \tag{B.2}$$

$$\frac{1}{T_P}\int_{t_\mu=0}^{t_\mu=T_P} I_{1,avg}\,\Delta i_1(t_\mu)\,dt_\mu \equiv 0 \tag{B.3}$$

$$I^2_{1,rms} = I^2_{1,avg} + \Delta I^2_{1,rms} \tag{B.4}$$

$$\begin{aligned} \frac{1}{(t_{\mu,i+1} - t_{\mu,i})}\int_{t_{\mu,i}}^{t_{\mu,i+1}} \Delta i_1^2(t_\mu)\,dt_\mu &= \frac{1}{3}(\Delta i^2_{1,i} + \Delta i_{1,i}\Delta i_{1,i+1} + \Delta i^2_{1,i+1}) \\ &= \frac{1}{3}\Delta i^2_{1,(i,i+1)} \end{aligned} \tag{B.5}$$

$$\Delta I^2_{1,rms} = \frac{1}{3}\sum_i \delta_{(i,i+1)}\Delta i^2_{1,(i,i+1)} \tag{B.6}$$

$$\delta_{(i,i+1)} = \frac{1}{T_P}(t_{\mu,i+1} - t_{\mu,i}) \tag{B.7}$$